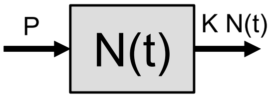
Introduction:

There are a number of very useful health physics applications to simple first order kinetics equations and their solutions.

These notes are based in part upon a lecture by Dr. Clay French entitled “Mind your P’s and K’s”.

General Problem

In Health Physics, one often encounters a problem of the form:



Where N(t) is the number of atoms in the compartment at time t
P is the production rate (atoms per unit time)
K is the overall removal constant from the compartment (per unit time).

If one assumes first order linear kinetics (for example, uniform instantaneous mixing), constant P, and N(t) = 0, then the general solution is of the form:

$$N(t) = \frac{P}{k}(1 - e^{-k \cdot t})$$

Another way to write this is:

$$N(t) = n\frac{(1 - e^{-k \cdot t})}{k \cdot t}$$

In this case, n = P t is the number of atoms produced in total
The second part of the function is the <u>survival fraction</u> at time t

If there is an acute injection of atoms, rather than a chronic source, there is no P function and the survival fraction at t is given by the familiar

$$N(t) = N(0) \cdot e^{-k \cdot t}$$

If you put N(0) atoms in a compartment and they are removed with an effective removal constant $k=k_1+k_2+\ldots$, then the number of atoms that will be removed by the nth process is given by:

$$U_n = \frac{k_n N(0)}{k}$$

Applications

The table on the next page summarizes the P and k for various commonly encountered situations.

References

1. Notes from a lecture by Dr. Clay French, Skrable Enterprises, 1993.
2. Cember, Introduction to Health Physics, 3rd Edition, McGraw-Hill, 1996.
3. Schlein, Health Physics and Radiological Health Handbook, Scinta, 1992.

Application	Production Rate (atoms per second)		Removal Constant k (per second)	
Internal Dose	$\dot{I}$	Atoms per minute intake, uptake, deposition etcetera	$\lambda_R + \lambda_B$	λ_R = Decay constant for i (s^{-1}) λ_B = Biological removal constant (s^{-1})
Air Sampling – Activity on Filter	$(\frac{U}{\lambda})FR$	U = Activity concentration (Bq/m^3) λ = Decay constant (s^{-1}) F = Pump flow rate ($m^3\ s^{-1}$) R = Filter retention factor (0 to 1)	λ_R	(the decay constant, s^{-1})
Fission Product Production	$w_{th}Y_i$	W_{th} = fission rate = 3.140E10 fissions s^{-1} watt^{-1}) Y_i = Yield (atoms per fission) for radionuclide i	$\lambda + \sigma_i\varphi$	λ = Decay constant for i (s^{-1}) σ_i = Removal cross section for i (cm^2) φ = Neutron fluence rate ($cm^{-2}s^{-1}$)
Ventilation	E	E = Emanation rate (atoms per second)	$\lambda + \frac{F}{v}$	λ= Decay constant (s^{-1}) F = Ventilation flow rate ($m^3\ s^{-1}$) v = Volume of space (m^3)
Neuron Activation	$\sigma_t\varphi N_t$	σ_t = activation cross section for target to i (cm^2) φ = Neutron fluence rate ($cm^{-2}s^{-1}$) N_t = Number of target atoms	$\lambda + \sigma_i\varphi$	λ = Decay constant for i (s^{-1}) σ_i = Removal cross section for i (cm^2) φ = Neutron fluence rate ($cm^{-2}s^{-1}$)

Example Problem :

What is the activity concentration in air if an air sample which is collected for T1 seconds, allowed to decay for T2 seconds and then counted for T3 seconds is observed to cause C_s counts on the counting system which has an efficiency E?

Let U = the activity concentration (Bq/m^3)
R = the filter retention fraction
F = pump flow rate ($m^3\ s^{-1}$)
λ = decay constant for the radionuclide (s^{-1})
E = the counting efficiency for the detector system (counts per transformation)

The expected number of counts is given by the product of the counting efficiency, the number of atoms collected, the fraction that survive to T1, the fraction that survive to T2 and the fraction that decay during counting time T3.

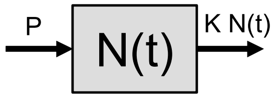
$$\langle C_s \rangle = E \cdot n \left[\frac{1 - e^{-\lambda \cdot T1}}{\lambda \cdot T1} \right] \cdot e^{-\lambda \cdot T2} (1 - e^{-\lambda \cdot T3})$$

$$n = P \cdot t = \frac{U \cdot R \cdot F}{\lambda} T1$$

$$\langle C_s \rangle = E \cdot \frac{U \cdot R \cdot F}{\lambda} T1 \cdot \left[\frac{1 - e^{-\lambda \cdot T1}}{\lambda \cdot T1} \right] \cdot e^{-\lambda \cdot T2} (1 - e^{-\lambda \cdot T3})$$

$$U = \frac{\lambda^2 \cdot C_s}{E \cdot R \cdot F \cdot (1 - e^{-\lambda \cdot T1}) e^{-\lambda \cdot T2} (1 - e^{-\lambda \cdot T3})}$$

Derivations of Solutions



Where N(t) is the number of atoms in the compartment at time t
P is the production rate (atoms per unit time) (constant with time)
K is the overall removal constant from the compartment (per unit time).

$$\begin{aligned}\frac{d\ N(t)}{dt} &= R_+ - R_- \\ &= P - K\ N(t)\end{aligned}$$

$$\frac{d\ N(t)}{dt} + K\ N(t) = P$$

Multiply throughout by e^{Kt}

$$\frac{d\ N(t)}{dt} e^{Kt} + K\ N(t) e^{Kt} = P e^{Kt}$$

But

$$\frac{d\ N(t)}{dt} e^{Kt} + K\ N(t) e^{Kt} = \frac{d\ (N(t) e^{Kt})}{dt}$$

So

$$\frac{d\ (N(t) e^{Kt})}{dt} = P e^{Kt}$$

And, integrating both sides,

$$\int \frac{d\ (N(t) e^{Kt})}{dt} dt = \int P e^{Kt} dt$$

$N(t) e^{Kt} = \int P e^{Kt} dt = P \int e^{Kt} dt = \frac{P}{K} e^{Kt} + C$ where C is the constant of integration

Dividing through by e^{Kt}

$N(t) = \frac{P}{K} + C e^{-Kt}$ (General Solution)

$N(t) = \frac{P}{K} - \frac{P}{K} e^{-Kt} = \frac{P}{K}(1 - e^{-K\,t})$ (When $N(0)=0$)

Boundary Conditions

$$N(0) = 0$$

$$0 = \frac{P}{K} + C e^{-K(0)}$$

$$C = -\frac{P}{K}$$

Fraction of Removal by any Route:

When there are i routes of removal, each with an associated rate constant k_i then

$$\begin{aligned} K &= k_1 + k_2 + \dots k_i \\ &= \sum_{1}^{i} k_i \end{aligned}$$

The total rate of removal from the compartment at any time and the rate by the ith route are given by:

$$\begin{aligned} R_- &= K\ N(t) \\ R_{-,i} &= k_i\ N(t) \end{aligned}$$

So, performing the definite integral for $t = 0$ to $t = \infty$, the total number of atoms leaving the compartment U and the number by the ith route, U_i, are given by.

$$U = K \int_0^\infty N(t)\ dt$$

$$U_i = k_i \int_0^\infty N(t)\ dt$$

so $\dfrac{U_i}{U} = \dfrac{k_i}{K}$

And for the case $N(0)=N_0$ and $P=0$

$$U = K \int_0^\infty N_0\ e^{-K\,t}\ dt = K\ \frac{N_0}{K} = N_0$$

$$U_i = k_i \int_0^\infty N_0\ e^{-K\,t}\ dt = k_i\ \frac{N_0}{K}$$