

# UNENE Graduate Course Reactor Thermal-Hydraulics Design

McMaster University  
Whitby

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Fuel-Coolant Heat Transfer

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# Outline

- General heat conduction equation
- Heat transfer in radial direction
- Heat transfer in axial direction
- Axial and radial temperature profiles
- Convective heat transfer and boiling
- Concept of dryout
- Conduction heat transfer coefficient

# General Heat Conduction Equation

- The interface between the fuel and the coolant is centrally important to reactor design because it limits the power output
- For a solid, the general energy thermal energy balance equation of an arbitrary volume

$$\iiint_V \frac{\partial(\rho e)}{\partial t} dV = \iiint_V q'''(\vec{r}, t) dV - \iint_S \vec{q}''(\vec{r}, t) \cdot \hat{n} ds$$

- Where  $\tilde{n}$  is the material density,  $e$  is the internal energy,  $V$  is the volume,  $S$  is the surface area,  $q'''$  is the volumetric heat generation,  $q''$  is the heat flux and  $n$  is the unit vector on the surface. We replace the internal energy with temperature,  $T$ , times the heat capacity  $c$ .

# General Heat Conduction Equation

- Using Gauss law, the heat balance equation is transformed into:

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\bar{r}, t) - \nabla \cdot q''(\bar{r}, t)$$

- Using Fourier law

$$q''(\bar{r}, t) = -k \nabla T(\bar{r}, t)$$

$$\frac{\partial(\rho c T)}{\partial t} = q'''(\bar{r}, t) - \nabla \cdot k \nabla T(\bar{r}, t)$$

$\rho$	kg/m <sup>3</sup>
$c$	J/(kg °K)
$k$	J/(m °K-sec)
$q''$	J/(m <sup>2</sup> -sec) = W/m <sup>2</sup>
$q'''$	J/(m <sup>3</sup> -sec) = W/m <sup>3</sup>
$T$	°K
$\alpha$	defined as $k/\rho c = \text{m}^2/\text{sec}$ .

# Heat Transfer in Radial Direction (Fuel)

$$\nabla \cdot k_f \nabla T(\bar{r}, t) = q'''(\bar{r}, t)$$

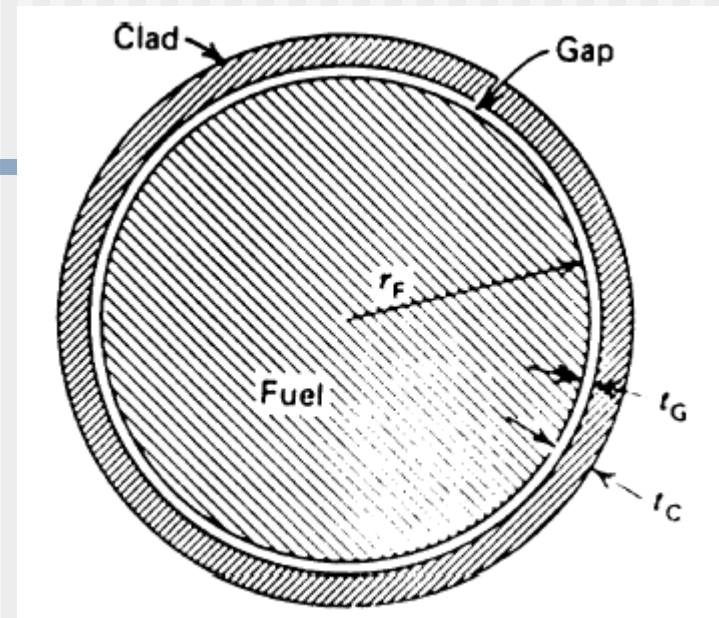
$$\frac{1}{r} \frac{d}{dr} \left( k_f r \frac{dT}{dr} \right) = q'''(r)$$

$$k_f r \frac{dT}{dr} = - \frac{r^2}{2} q'''(r)$$

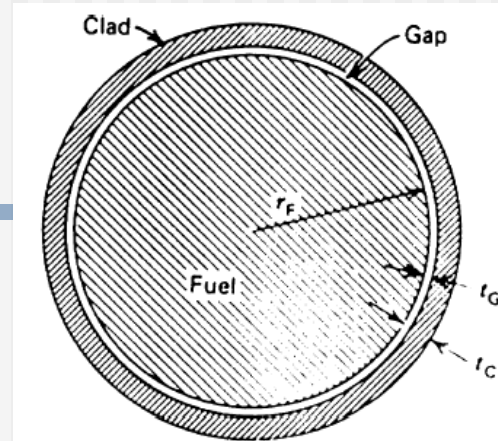
$$\int_{T_0}^{T_F} k_f(T) dT = - \frac{r_F^2}{4} q''' \equiv \bar{k}_f (T_F - T_0)$$

$$\begin{aligned} \Delta T_{fuel} \equiv T_0 - T_F &= \frac{r_F^2}{4k_f} q''' \\ &= \frac{q'}{4\pi k_f} \end{aligned}$$

where  $q' \equiv \pi r_F^2 q''' =$  linear power density



# Heat Transfer in Radial Direction (Gap)



$$\frac{1}{r} \frac{d}{dr} \left( k_G r \frac{dT}{dr} \right) = 0$$

$$k_G r \frac{dT}{dr} = \text{constant}$$

$$-k_G \frac{dT}{dr} \Big|_{r=r_F} = q'' = \frac{q'}{2\pi r_F}$$

$$k_G r \frac{dT}{dr} = -\frac{q'}{2\pi}$$

$$\therefore k_G \frac{dT}{dr} = -\frac{q'}{2\pi r}$$

$$k_G \Delta T_{GAP} = k_G (T_F - T_C) = \frac{q'}{2\pi} \ln \left( \frac{r_F + t_G}{r_F} \right)$$

$$\begin{aligned} \therefore \Delta T_{GAP} &= \frac{q'}{2\pi k_G} \ln \left( \frac{r_F + t_G}{r_F} \right) \\ &\approx \frac{q'}{2\pi r_F} \left( \frac{t_G}{k_G} \right) \quad \text{since } \ln(1+x) \approx x \end{aligned}$$

$$h_G (\Delta T_{GAP}) = q''$$

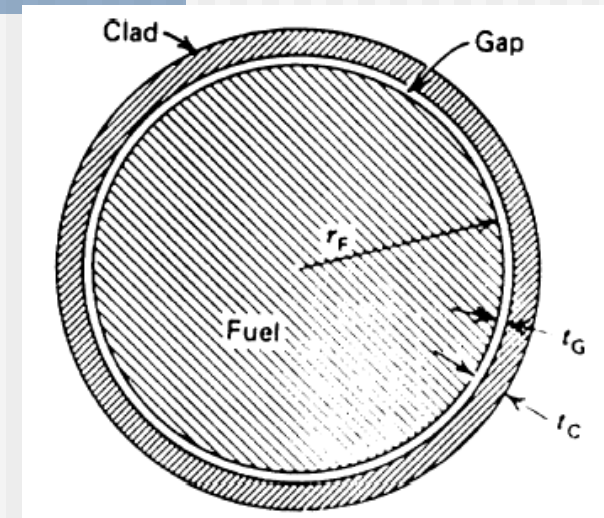
$$\Delta T_{GAP} = \frac{q'}{2\pi r_F h_G}$$

# Heat Transfer in Radial Direction (Clad)

$$\frac{1}{r} \frac{d}{dr} \left( k_C r \frac{dT}{dr} \right) = 0$$

$$k_C \Delta T_{CLAD} = k_C (T_C - T_S) = \frac{q'}{2\pi} \ln \left( \frac{r_F + t_G + t_C}{r_F + t_G} \right)$$

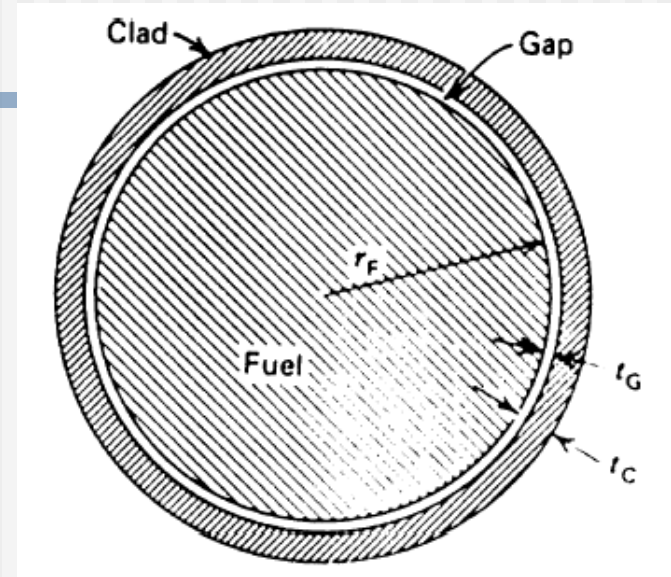
$$\begin{aligned} \therefore \Delta T_{CLAD} &= \frac{q'}{2\pi k_C} \ln \left( \frac{r_F + t_G + t_C}{r_F + t_G} \right) \\ &\approx \frac{q'}{2\pi (r_F + t_G)} \left( \frac{t_G + t_C}{k_C} \right) \quad \text{since } \ln(1+x) \approx x \end{aligned}$$



# Heat Transfer in Radial Direction (Coolant)

$$q'' = h_S (T_S - T_{FL})$$

$$\Delta T_{COOL} = \frac{q'}{2\pi h_S (r_F + t_C + t_G)}$$



$$T_0 - T_{FL} = \frac{q'}{2\pi} \left( \frac{1}{2k_f} + \frac{1}{h_G r_F} + \frac{t_G + t_C}{k_C (r_F + t_G)} + \frac{1}{h_S (r_F + t_G + t_C)} \right)$$

# Heat Transfer in Axial Direction

AXIAL VARIATION OF NEUTRON FLUX:

$$\phi = \phi_{\max} \cos\left(\frac{\pi z}{H}\right) \quad H - \text{effective core length}$$

AXIAL HEAT GENERATION FOLLOWS THE NEUTRON FLUX VARIATION

$$q''' = q'''_{\max} \cos\left(\frac{\pi z}{H}\right)$$

HEAT BALANCE ALONG A FUEL ELEMENT

$$w c_p dT_f = q''' A_f dz$$

$w$  [kg/s] Mass flow rate

$c_p$  [kJ/kg °C] specific heat

$dT_f$  [°C] coolant temp rise between  $z$  and  $z+dz$

$A_c$  [m<sup>2</sup>] cross-section area of one fuel element

# Heat Transfer in Axial Direction

$$w C_p \int_{T_{f0}}^{T_f} dT_f = \dot{q}_{\max} A_f \int_{-H/2}^z \cos\left(\frac{\pi z}{H}\right) dz$$

$$T_f = T_{f0} + \frac{\dot{q}_{\max} A_f H}{\pi w C_p} \left[ 1 + \sin\left(\frac{\pi z}{H}\right) \right]$$

$$= T_{f0} + \frac{\dot{q}_{\max} V_f}{\pi w C_p} \left[ 1 + \sin\left(\frac{\pi z}{H}\right) \right]$$

$V_f$  [m<sup>3</sup>] Volume of the fueled portion of the fuel element.

$$T_{f\max} = T_{f0} + \frac{2 \dot{q}_{\max} V_f}{\pi w C_p} \quad \text{at } z = H/2$$

# Heat Transfer in Axial Direction

THE HEAT BALANCE BETWEEN SHEATH AND COOLANT LEADS TO THE FOLLOWING EQUATIONS FOR SHEATH TEMP :

$$h C_c (T_c - T_f) dz = \dot{q}_{\max} A_f \cos\left(\frac{\pi z}{H}\right) dz$$

$C_c$  [m] Circumference of the fuel sheath

$T_c$  [°C] Sheath temp.

$h$  [kJ/m<sup>2</sup>°C] Heat transfer coefficient

$$T_c = T_{f0} + \frac{\dot{q}_{\max} V_f}{\pi W C_p} \left[ 1 + \sin\left(\frac{\pi z}{H}\right) \right] + \frac{\dot{q}_{\max} A_f}{h C_c} \cos\left(\frac{\pi z}{H}\right)$$

# Heat Transfer in Axial Direction

Maximum sheath temperature:

$$\frac{dT_c}{dz} = 0 \quad Z_{\max} = \frac{H}{\pi} \tan^{-1} \left( \frac{h_c c_p H}{\pi W c_p} \right)$$
$$= \frac{H}{\pi} \cot^{-1} (\pi W c_p R_h)$$
$$R_h = \frac{1}{hA} = \frac{1}{h_c c_p H}$$

Maximum fuel temperature:

$$\frac{T_m - T_f}{R H} = q_{\max}''' A_f \cos\left(\frac{\pi z}{H}\right)$$
$$T_m = T_{f0} + \frac{q_{\max}''' V_f}{\pi W c_p} \left[ 1 + \sin\left(\frac{\pi z}{H}\right) \right] + q_{\max}''' V_f R \cos\left(\frac{\pi z}{H}\right)$$

$R$  = Total thermal resistance across the fuel element

# Heat Transfer in Axial Direction

Maximum fuel temperature:

$$\frac{dT_{fm}}{dz} = 0 \quad z_{fm \max} = \frac{H}{\pi} \cot^{-1}(\pi w c_p R)$$

Maximum fuel temperature:

$$T_{fm \max} = T_{f0} + q_{\max}''' V_f R \left[ \frac{1 + \sqrt{1 + \beta^2}}{\beta} \right]$$

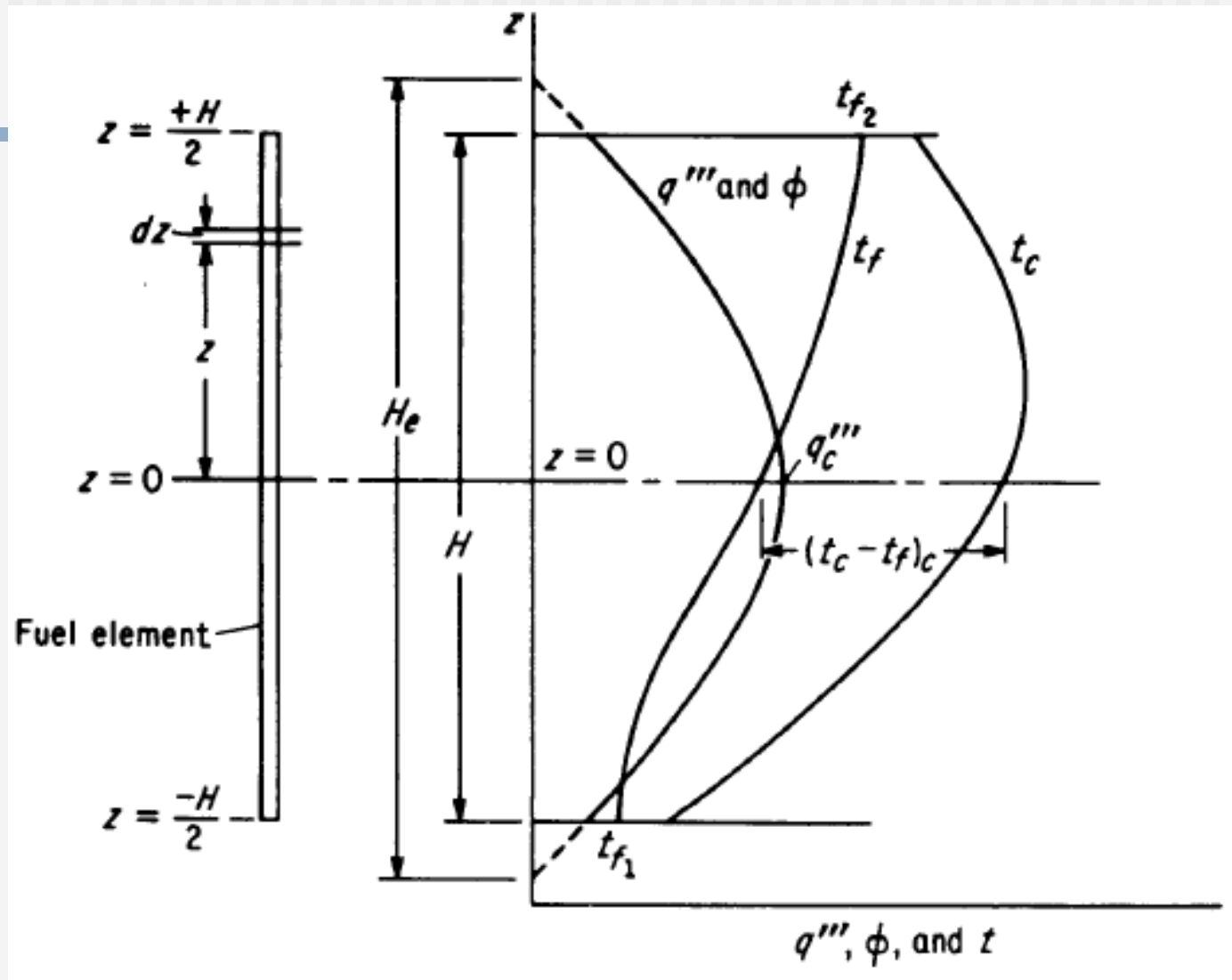
$$\beta = \pi w c_p R$$

Maximum sheath temperature:

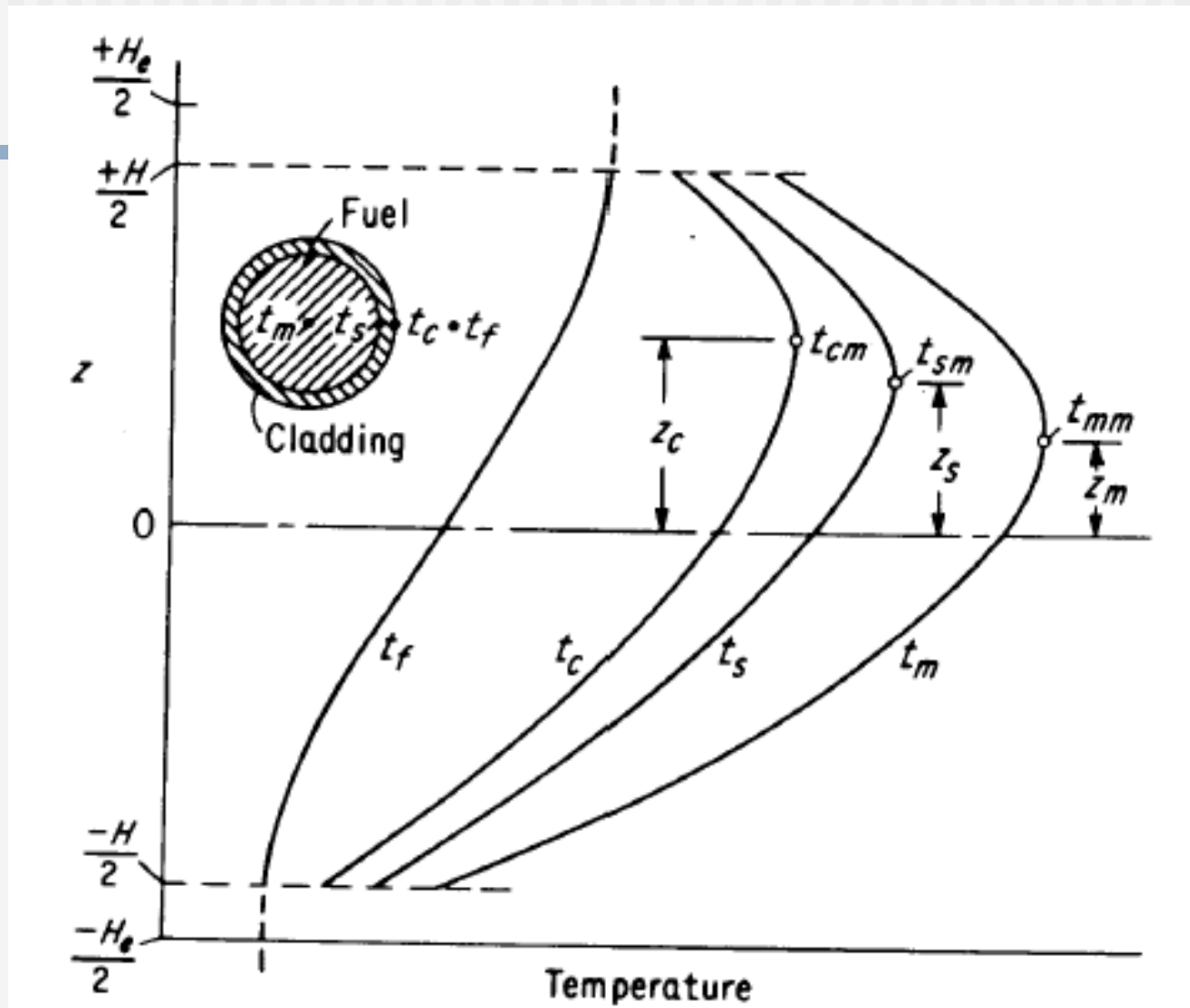
$$T_{c \max} = T_{s0} + q_{\max}''' V_s R_h \left[ \frac{1 + \sqrt{1 + \alpha^2}}{\alpha} \right]$$

$$\alpha = \pi w c_p R_h$$

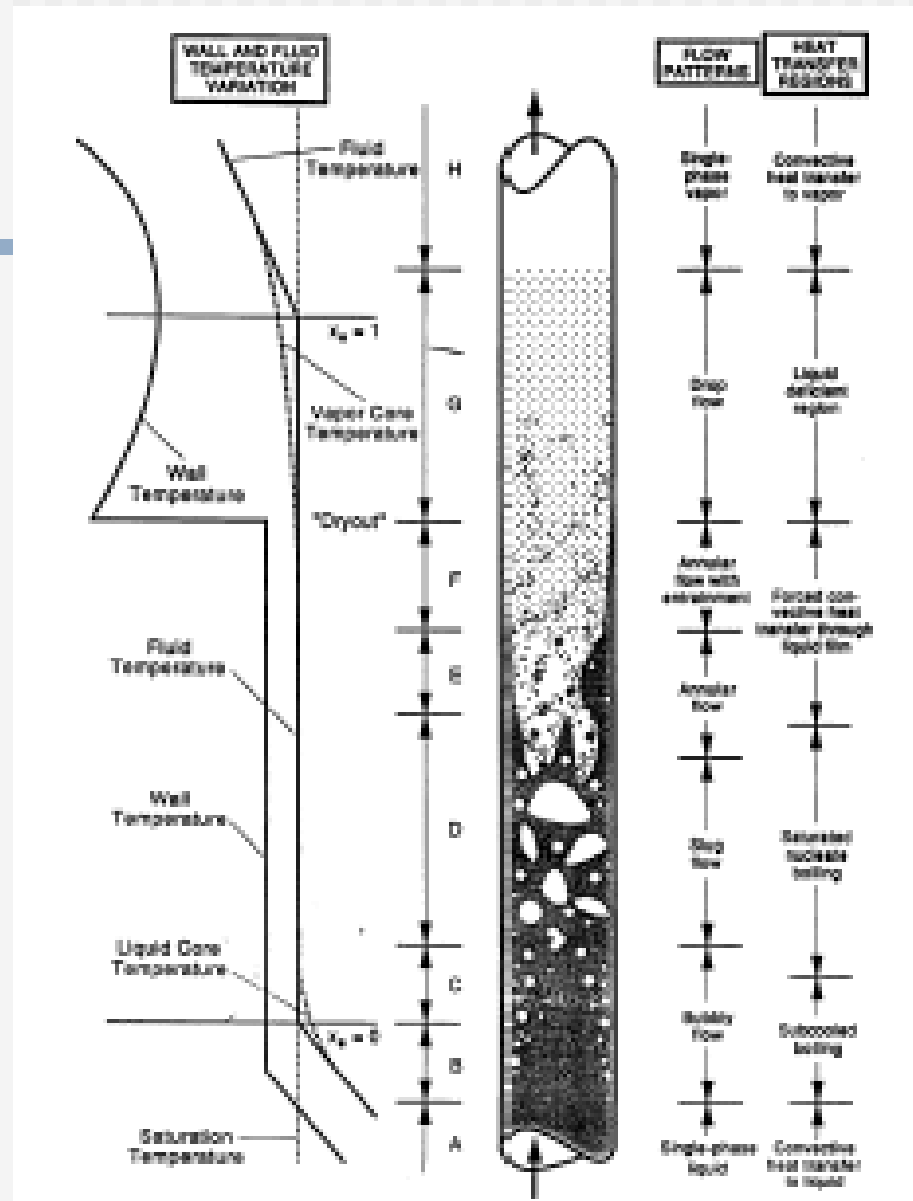
# Axial and Radial Temperature Profiles



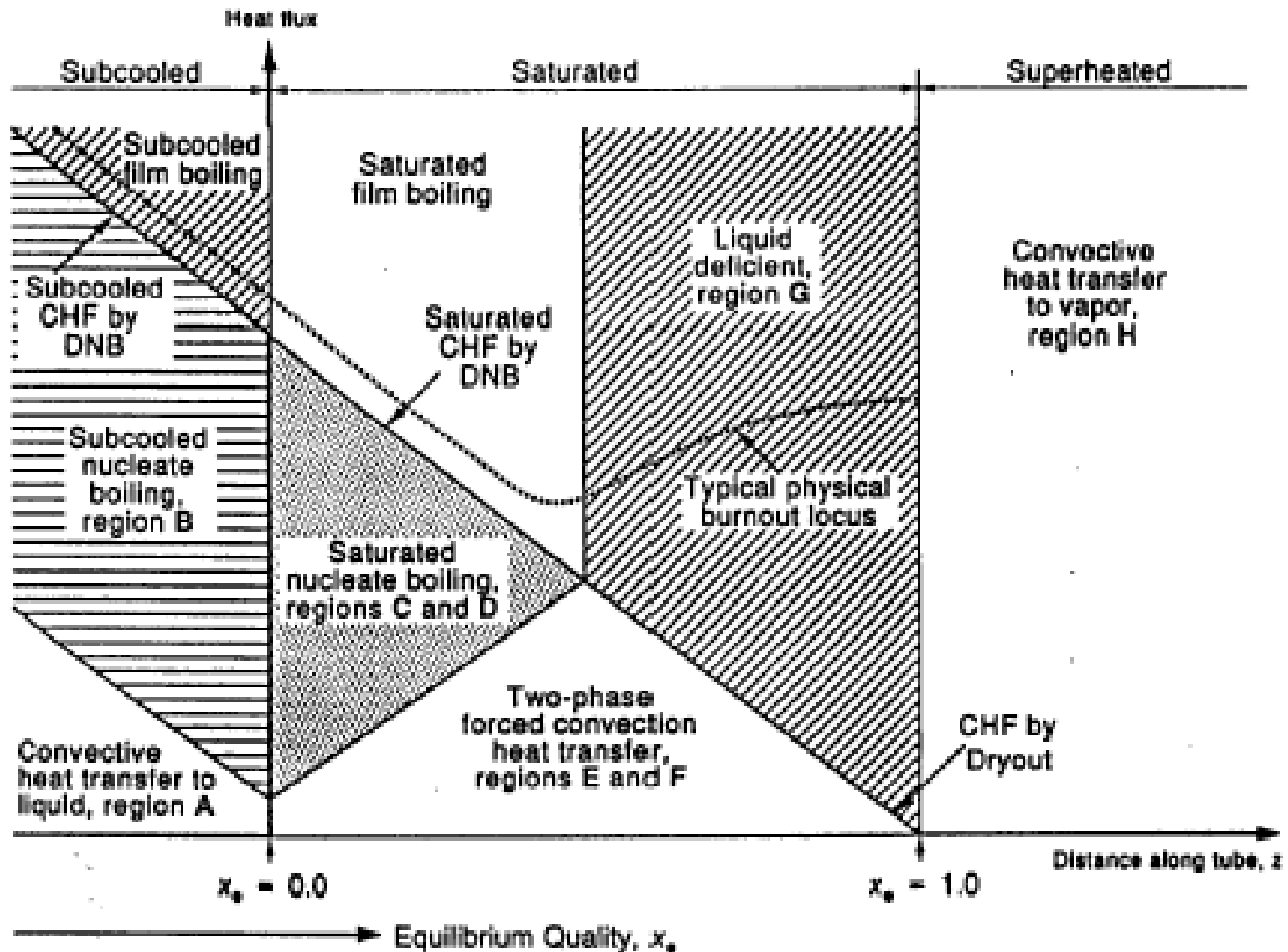
# Axial and Radial Temperature Profiles



# Convective Heat Transfer and Boiling



# Concept of Dryout



# Conduction Heat Transfer Coefficient

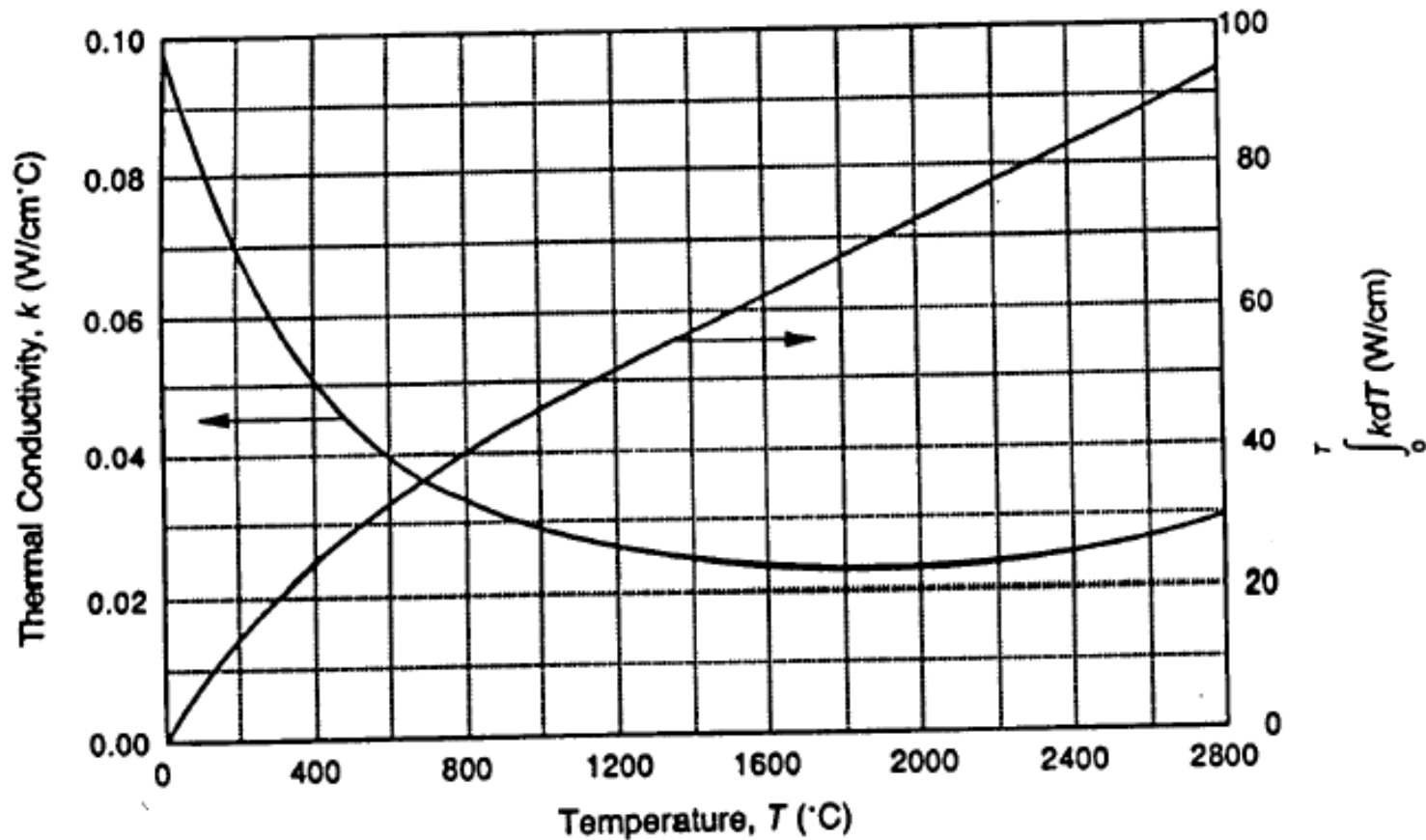


Figure 8-1 Thermal conductivity of  $\text{UO}_2$  at 95% density from Lyon. (From Hann et al. [9].)

# Conduction Heat Transfer Coefficient

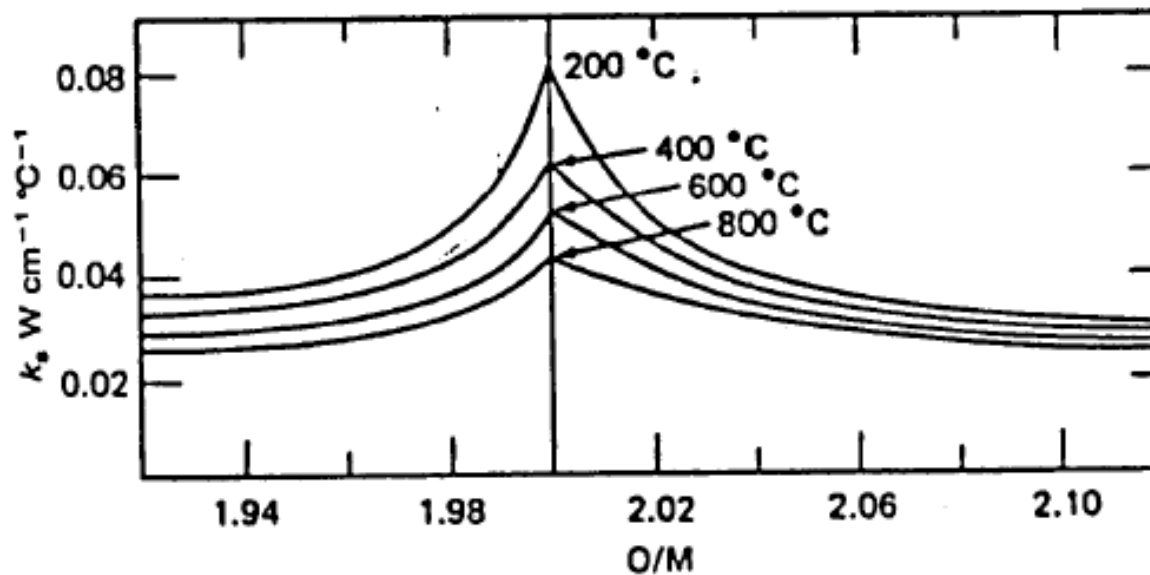


Figure 8-3 Thermal conductivity of  $\text{UO}_{0.8}\text{Pu}_{0.2}\text{O}_{2-x}$  as a function of the O/(U + Pu) ratio. (From Schmidt and Richter [24].)

# Conduction Heat Transfer Coefficient

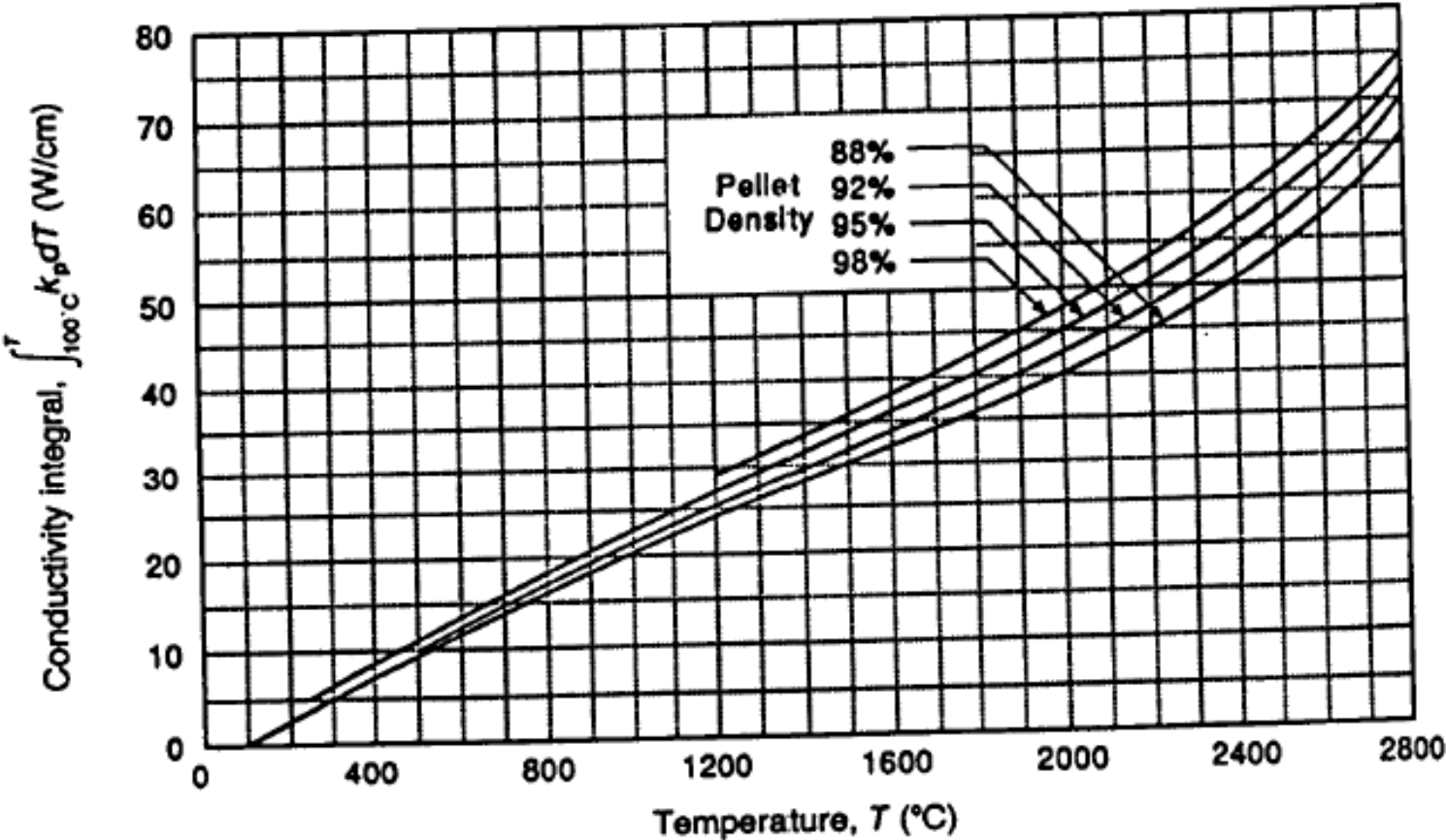
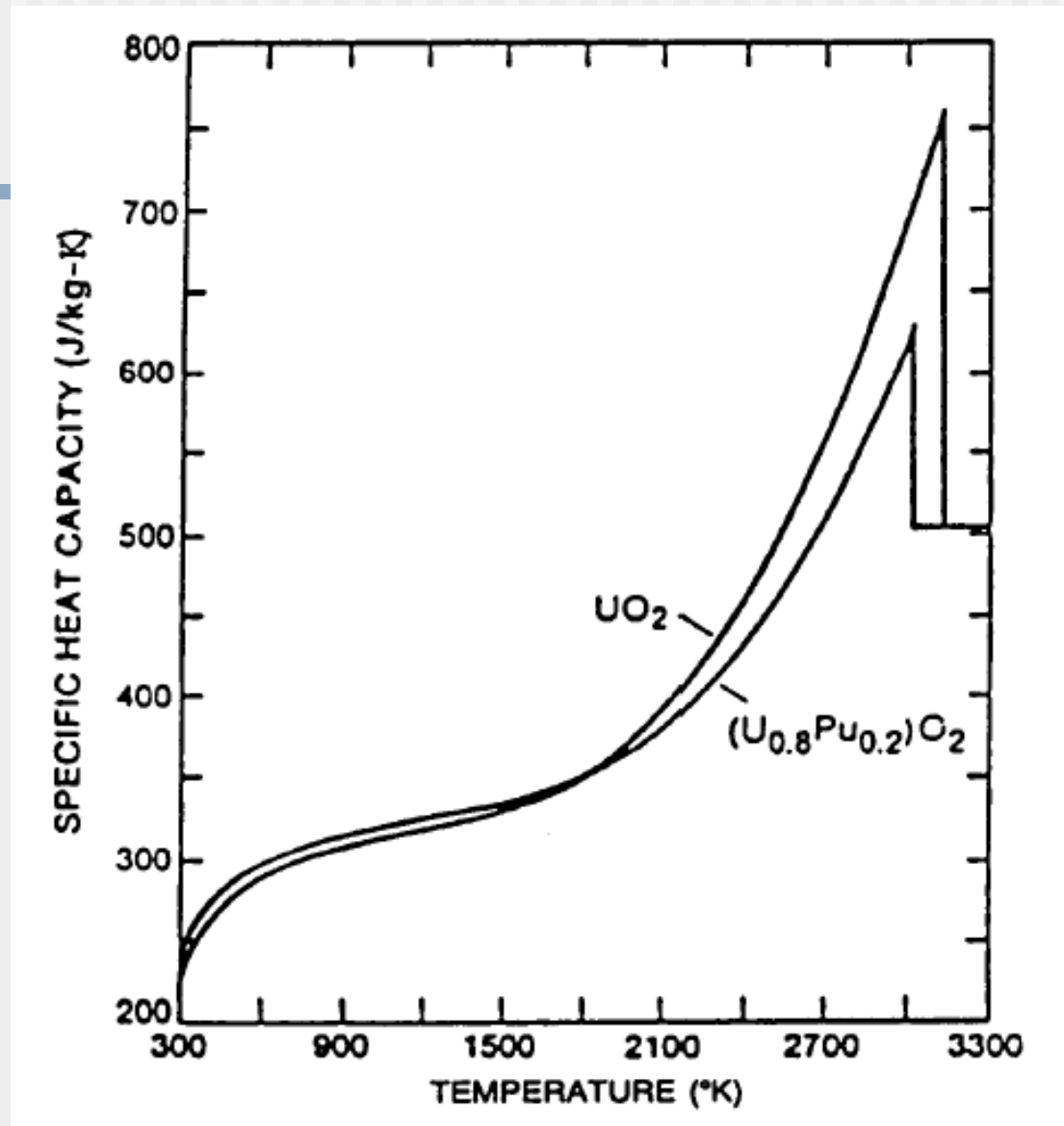
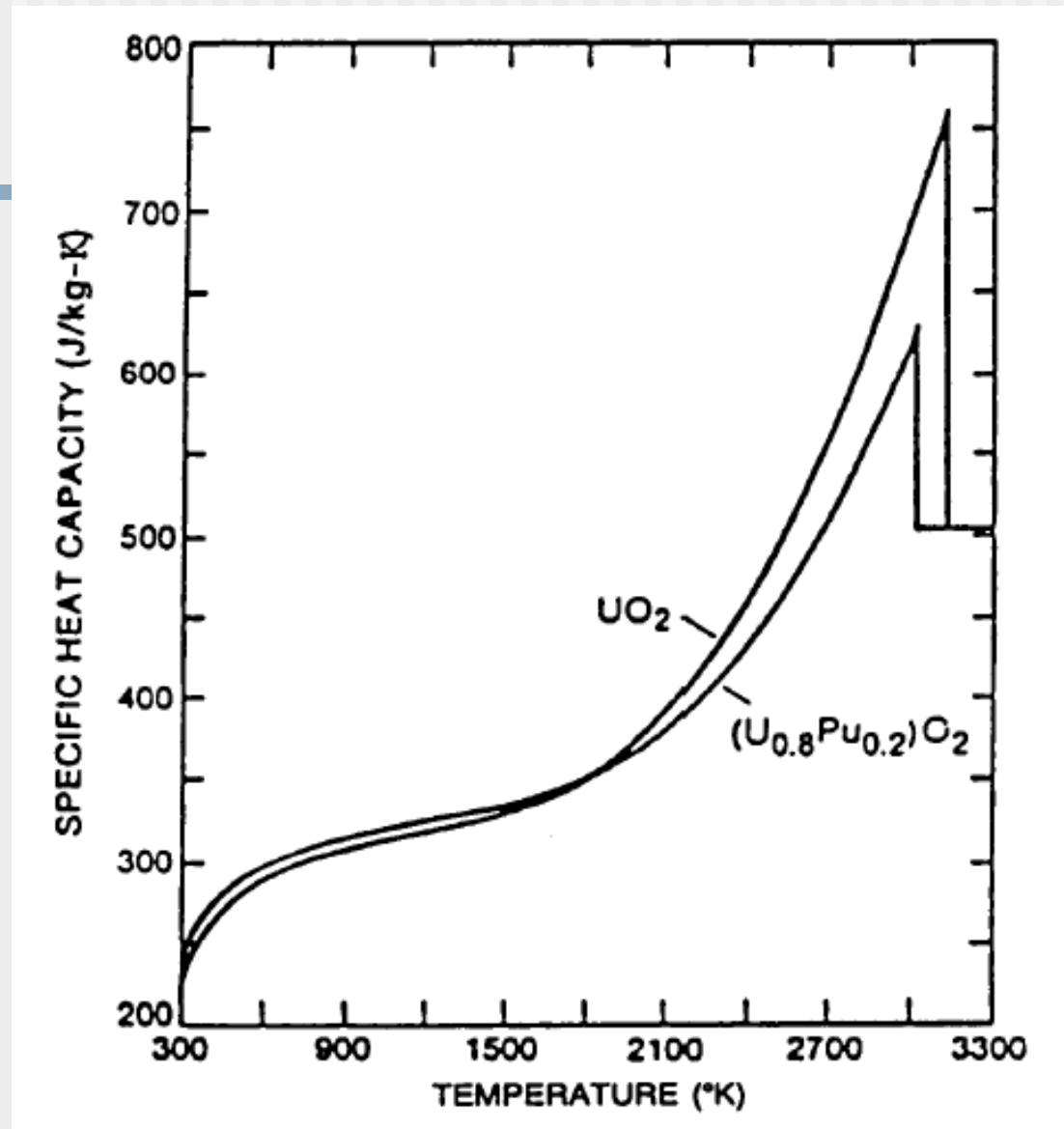


Figure 8-2  $\int_{100^{\circ}\text{C}}^T k_p dT$  versus temperature for mixed oxide fuel. Note: 32.8 W/cm = 1 kW/ft.

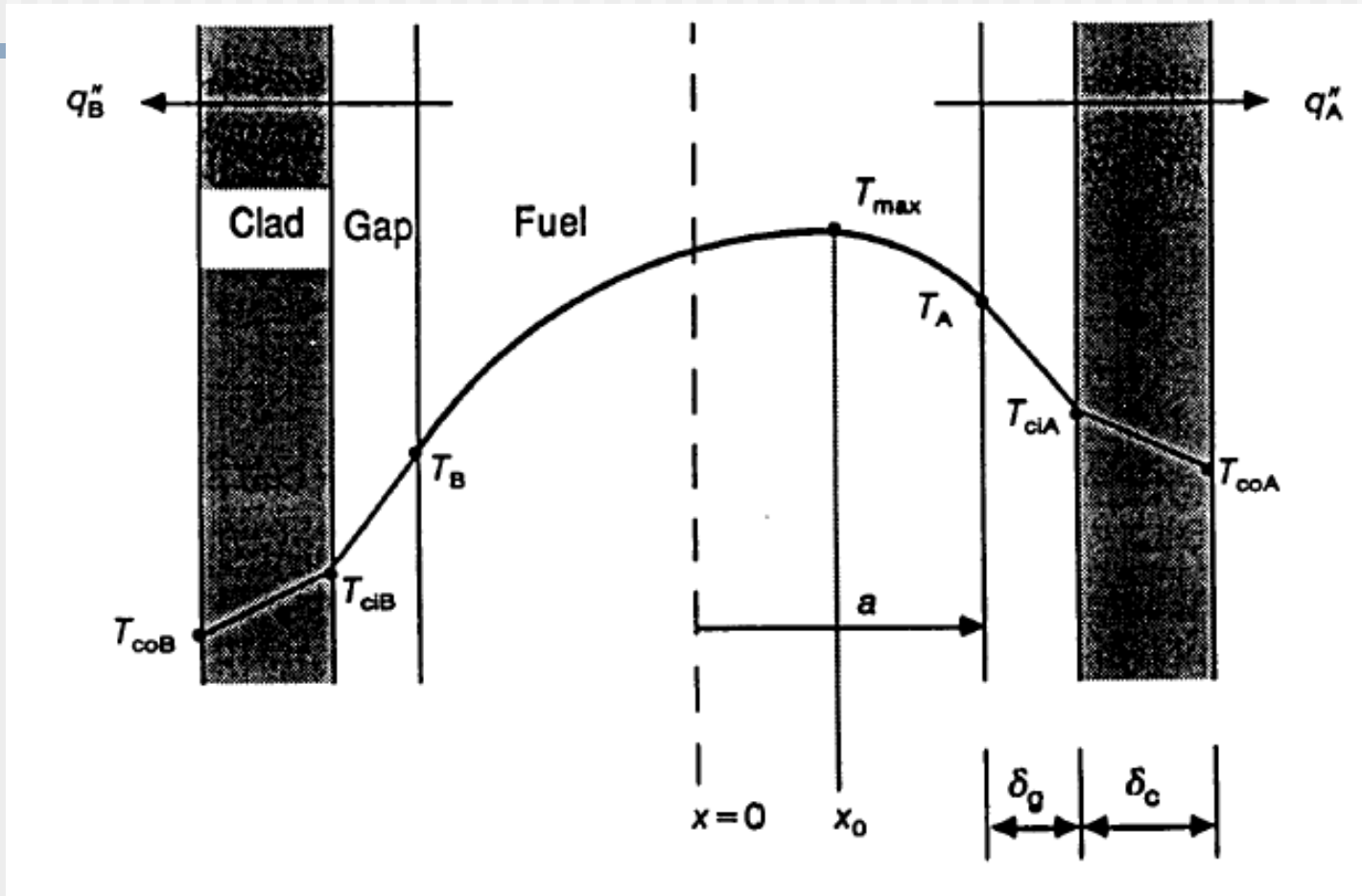
# Conduction Heat Transfer Coefficient



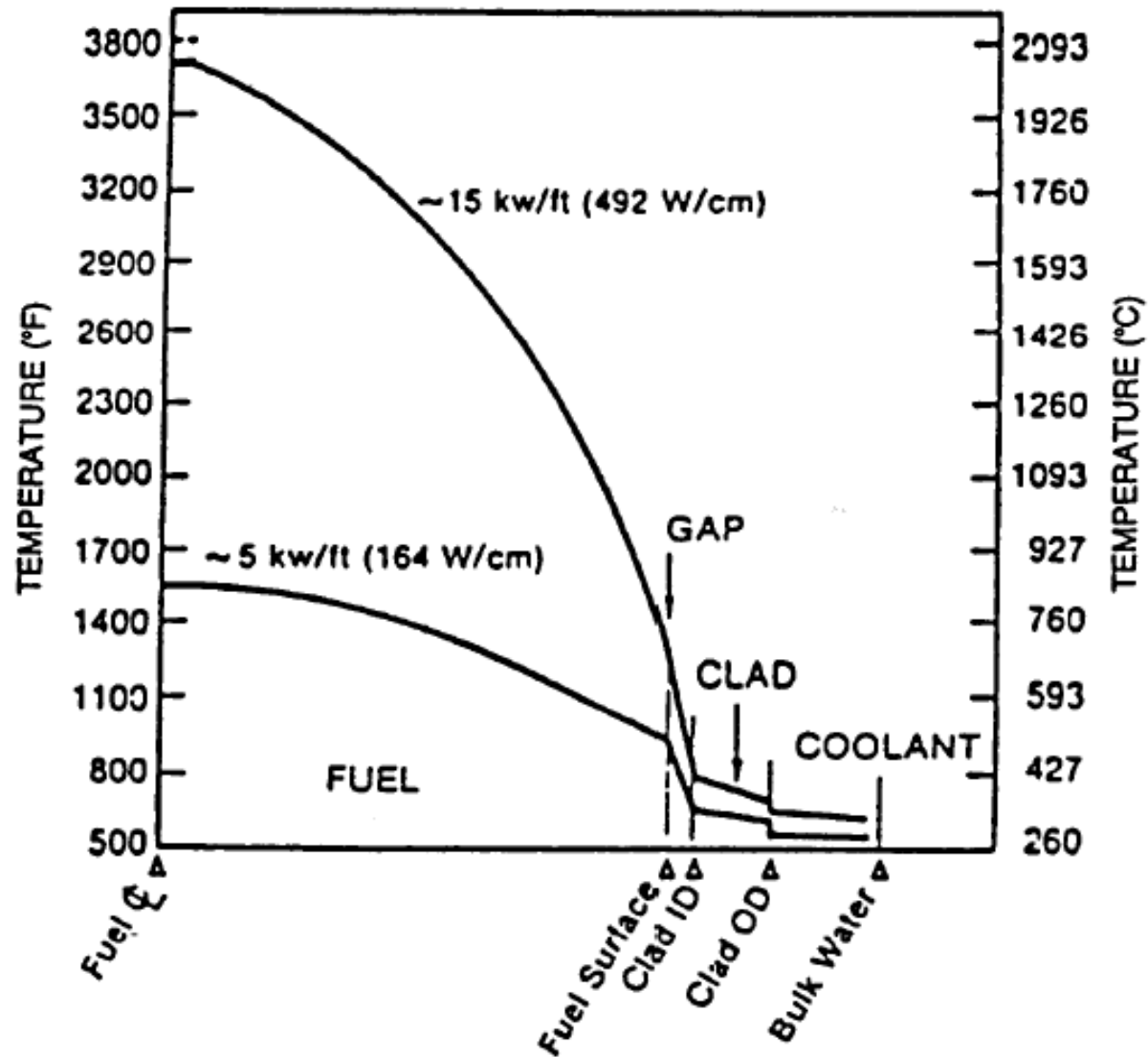
# Conduction Heat Transfer Coefficient



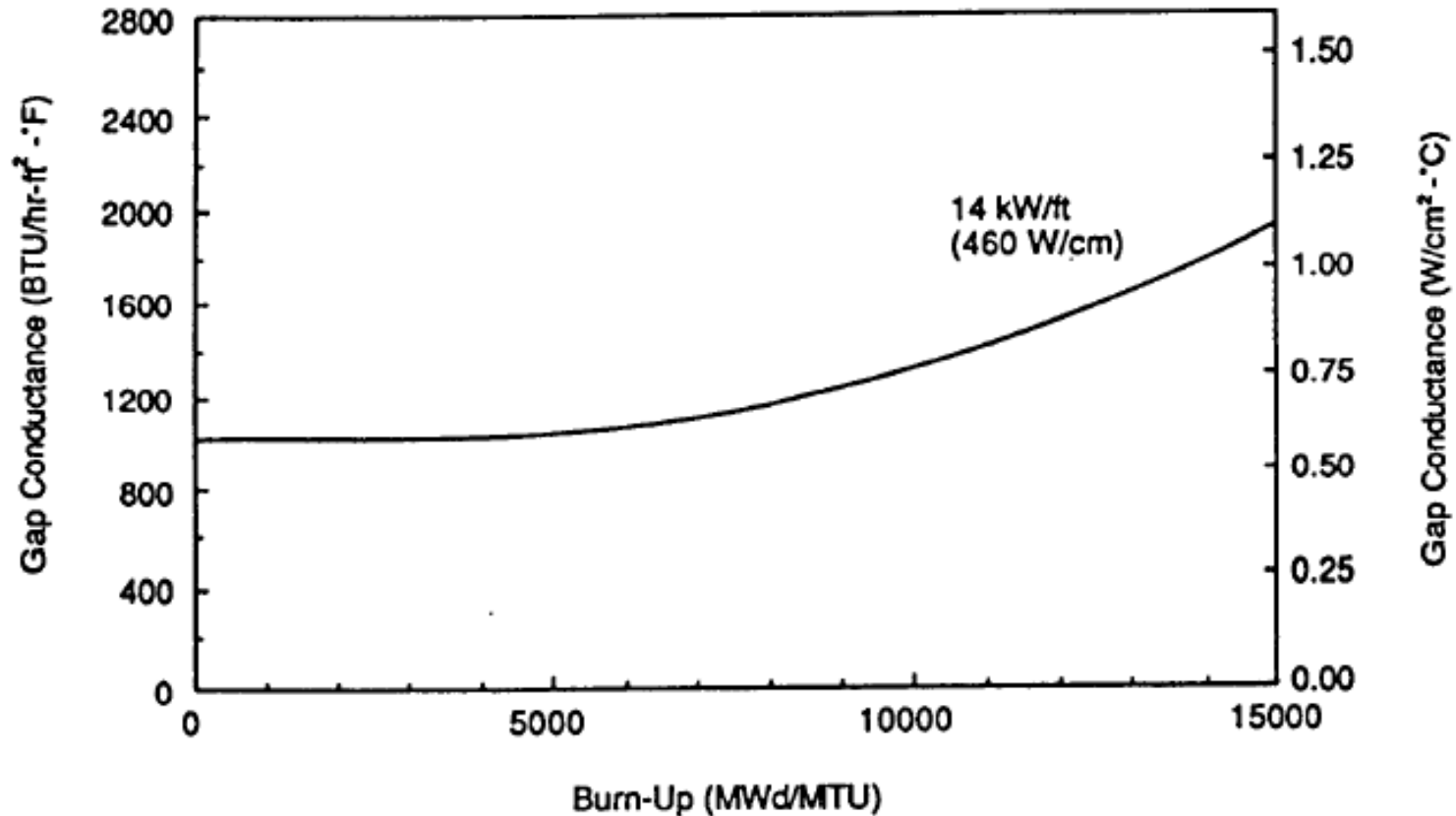
# Temperature Profiles



# Temperature Profiles

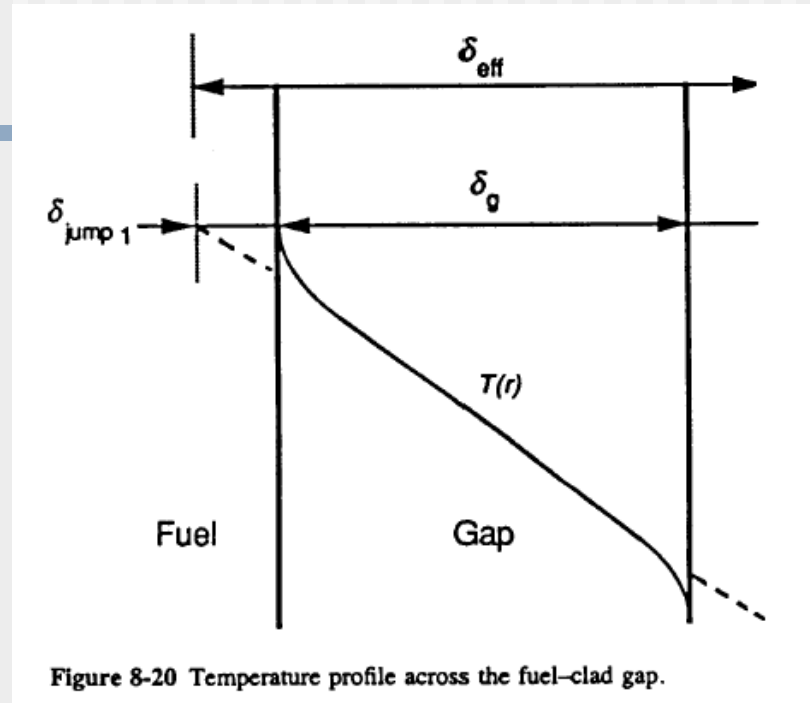
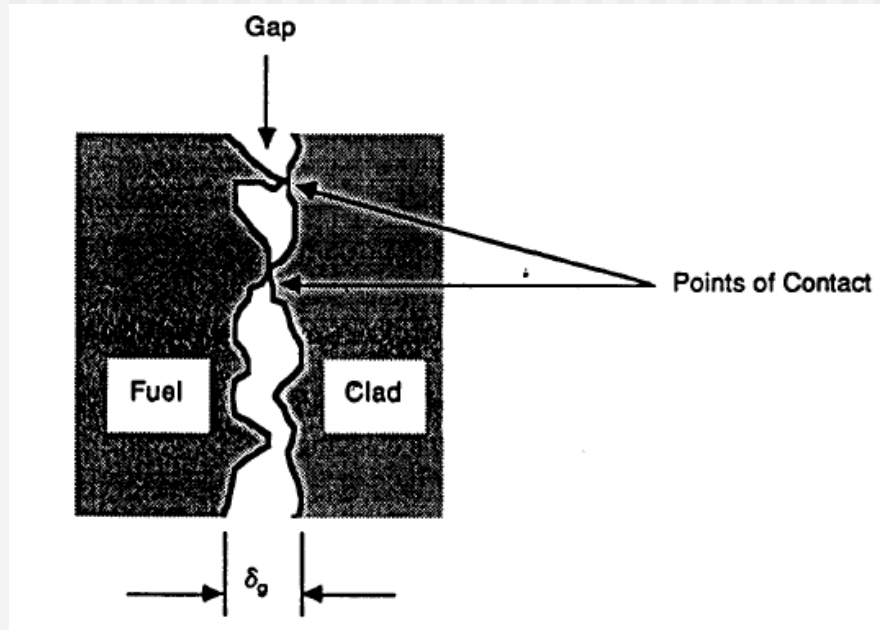


# Temperature Profiles



**Figure 8-19** Variations of gap conductance with burnup for a PWR fuel rod (pressurized with helium) and operating at 14 kW/ft (460 W/cm). (From Fenech [6].)

# Temperature Profiles



# Temperature Profiles

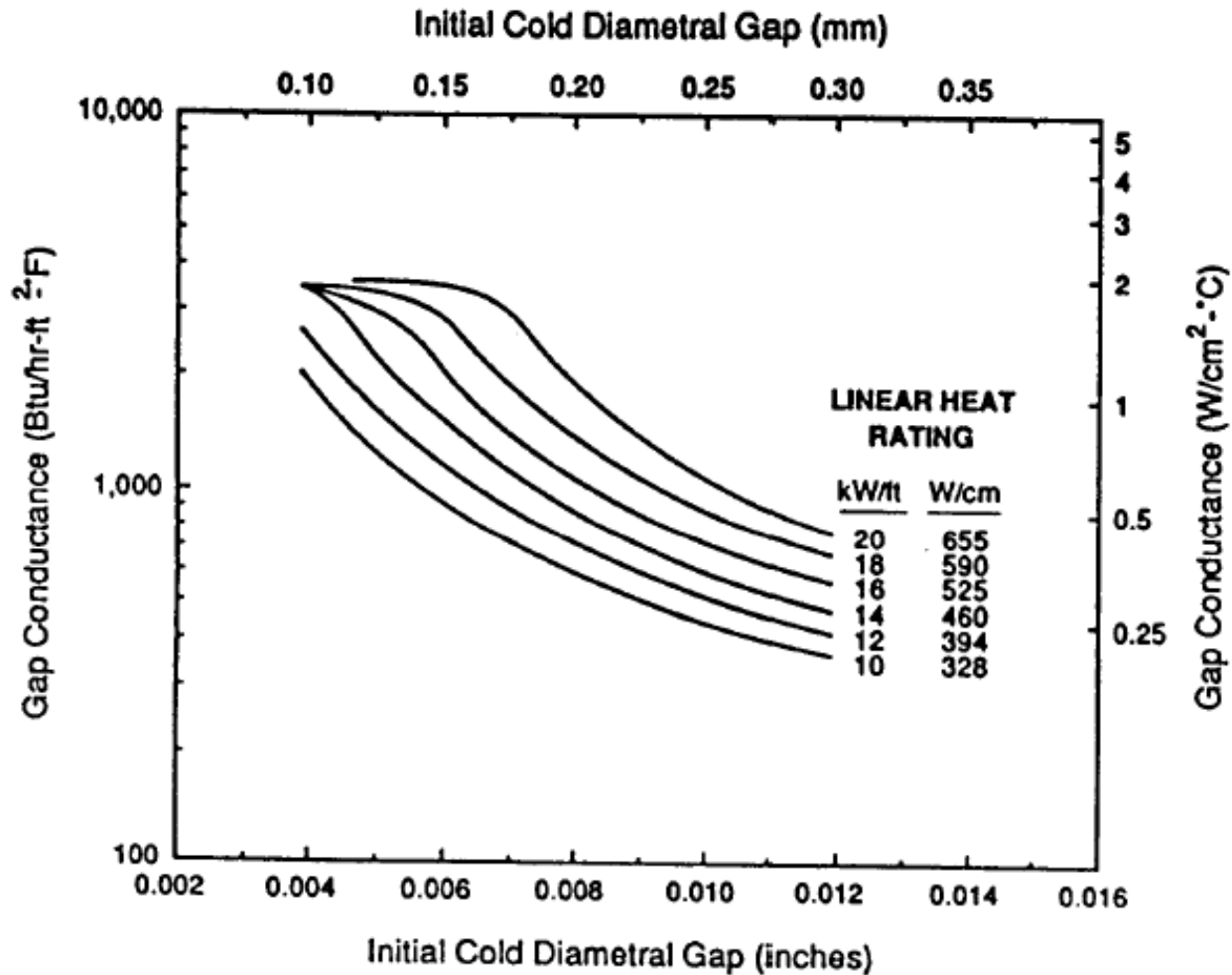


Figure 8-22 Calculated gap conductance as a function of cold diametral gap in a typical LWR fuel rod. (From Horn and Panisko, [10].)

# Questions?

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