

CLADDING CONDUCTION

FLAT PLATE CONDUCTION

CONDUCTION EQUATION

$$Q_x = -kA \frac{dT}{dx}$$

GENERAL EQUATION

$$\frac{d^2T}{dx^2} = 0$$

$$\frac{dT}{dx} = C_1$$

$$T = C_1 x + C_2$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad x = x_1$$

$$T = T_2 \quad \text{AT} \quad x = x_2$$

THUS : $T_1 = C_1 x_1 + C_2$ HENCE C_2

$$T_2 = C_1 x_2 + C_2$$

$$T_2 - T_1 = C_1 (x_2 - x_1) \quad \text{HENCE } C_1$$

TEMPERATURE PROFILE

$$T = C_1 x + C_2$$

$$= \left(\frac{T_2 - T_1}{x_2 - x_1} \right) x + T_1 - \left(\frac{T_2 - T_1}{x_2 - x_1} \right) x_1$$

$$= T_1 + \left(\frac{T_2 - T_1}{x_2 - x_1} \right) (x - x_1)$$

$$= T_1 + \left(\frac{x - x_1}{x_2 - x_1} \right) (T_2 - T_1)$$

HEAT CONDUCTED

$$\begin{aligned} Q_x &= -kA \frac{dT}{dx} \\ &= -kA c_1 \\ &= -kA \frac{T_2 - T_1}{x_2 - x_1} \\ \frac{Q_x}{A} &= -k \left(\frac{T_2 - T_1}{x_2 - x_1} \right) \\ q_x &= -k \left(\frac{T_2 - T_1}{x_2 - x_1} \right) \end{aligned}$$

NOTE THAT AREA A IS CONSTANT FOR FLAT PLATE
BUT NOT FOR CYLINDRICAL AND SPHERICAL SHELLS

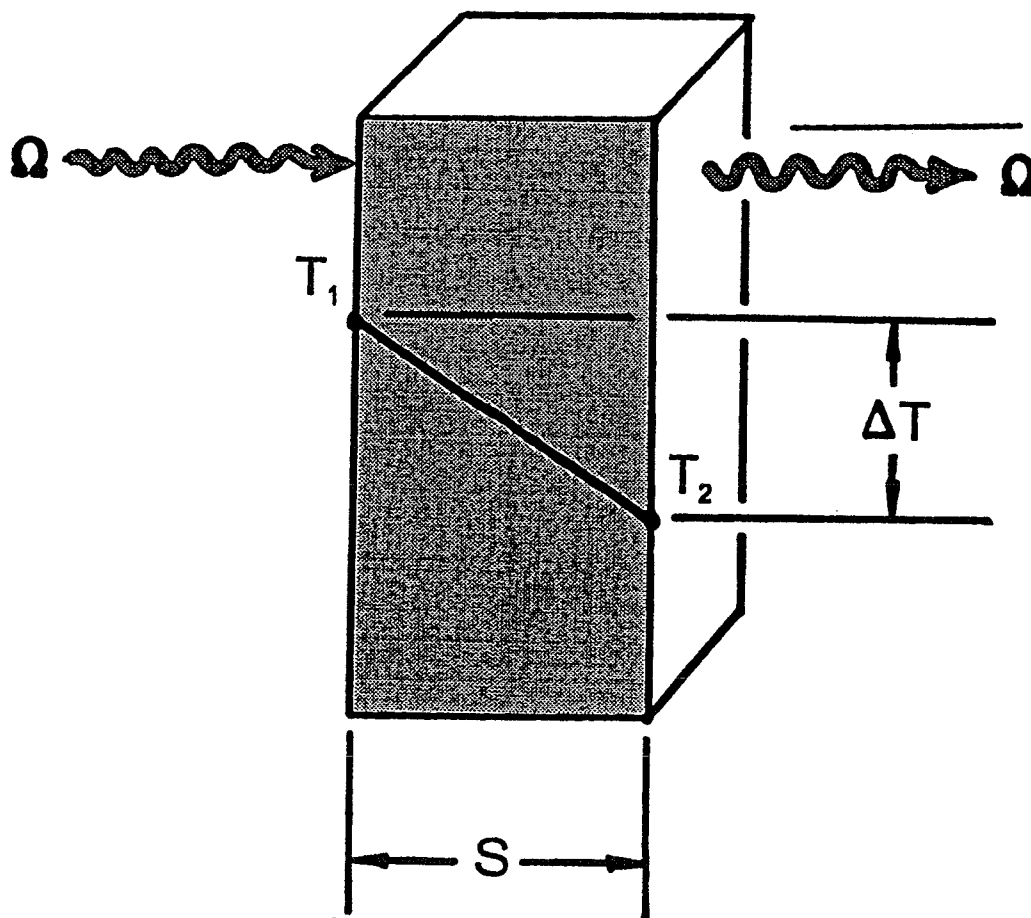


Figure 1 Heat conduction through a solid flat slab

CYLINDRICAL SHELL CONDUCTION

CONDUCTION EQUATION

$$Q_r = -kA_r \frac{dT}{dr}$$

GENERAL EQUATION

$$\frac{d^2T}{dr^2} + \frac{1}{r} \frac{dT}{dr} = 0$$

$$r \frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) = 0$$

$$r \frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T = C_1 \ln r + C_2$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad r = r_1$$

$$T = T_2 \quad \text{AT} \quad r = r_2$$

THUS $T_1 = C_1 \ln r_1 + C_2$ HENCE C_2

$$T_2 = C_1 \ln r_2 + C_2$$

$$T_2 - T_1 = C_1 \ln r_2 - C_1 \ln r_1$$

$$T_2 - T_1 = C_1 \ln \frac{r_2}{r_1} \quad \text{HENCE } C_1$$

TEMPERATURE PROFILE

$$\begin{aligned} T &= C_1 \ln r + C_2 \\ &= \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) \ln r + T_1 - \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) \ln r_1 \\ &= T_1 + \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) (\ln r - \ln r_1) \\ &= T_1 + \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) \ln r / r_1 \end{aligned}$$

HEAT CONDUCTED

$$\begin{aligned} Q_r &= -k A_r \frac{dT}{dr} \quad \text{AREA VARIES WITH } r \\ &= -k 2\pi r L \frac{C_1}{r} \\ &= -k 2\pi r \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) / r \\ &= -k 2\pi L \left(\frac{T_2 - T_1}{\ln r_2 / r_1} \right) \end{aligned}$$

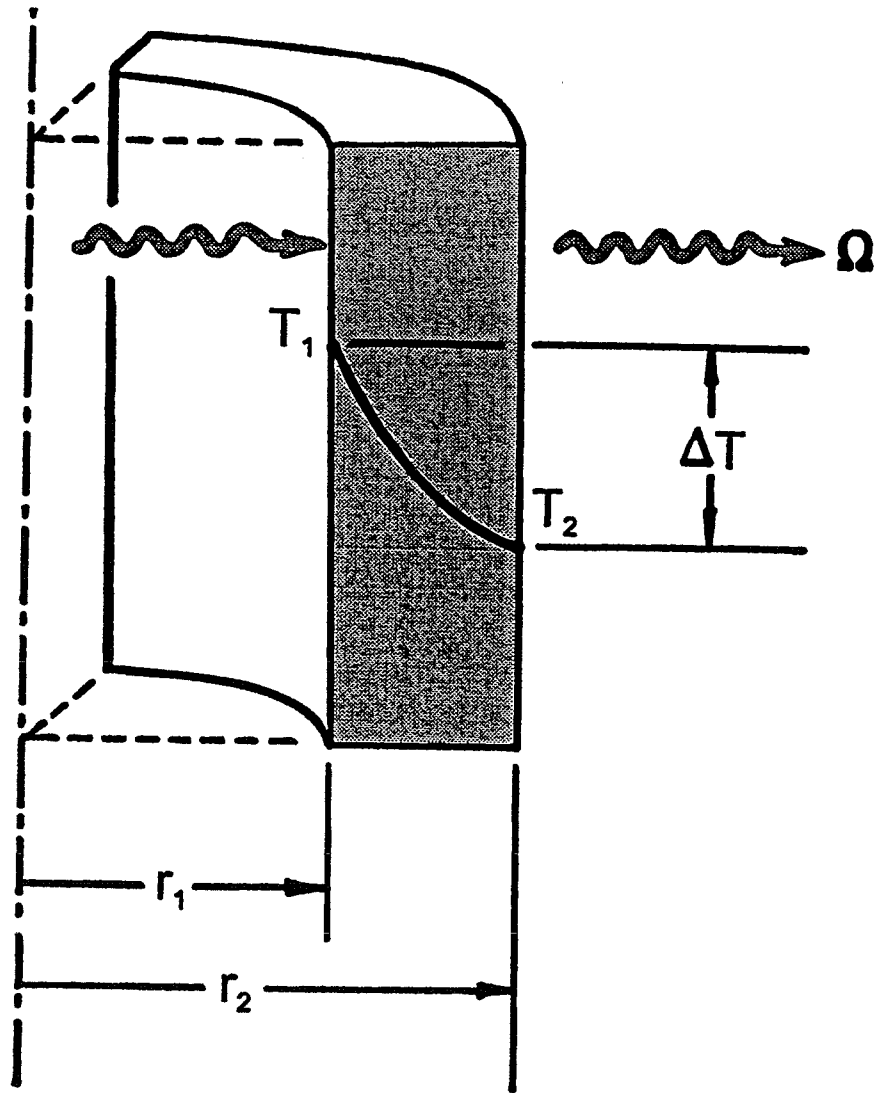


Figure 3 Heat conduction through a solid cylindrical wall

SPHERICAL SHELL CONDUCTION

CONDUCTION EQUATION

$$Q_{r \rightarrow} = -kA_r \frac{dT}{dr}$$

GENERAL EQUATION

$$\frac{d^2T}{dr^2} + \frac{2}{r} \frac{dT}{dr} = 0$$

$$r \frac{d^2T}{dr^2} + 2 \frac{dT}{dr} = 0$$

$$\frac{d}{dr} \left(r \frac{dT}{dr} \right) + \frac{dT}{dr} = 0$$

$$r \frac{dT}{dr} + T = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r} - \frac{T}{r}$$

$$T r = C_1 r + C_2$$

$$T = C_1 + \frac{C_2}{r}$$

$$C_1 - T = -\frac{C_2}{r}$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad r = r_1$$

$$T = T_2 \quad \text{AT} \quad r = r_2$$

THUS: $T_1 = C_1 + \frac{C_2}{r_1}$ HENCE C_1

$$T_2 = C_1 + \frac{C_2}{r_2}$$

$$T_2 - T_1 = \frac{C_2}{r_2} - \frac{C_2}{r_1}$$

$$T_2 - T_1 = C_2 \left(\frac{1}{r_2} - \frac{1}{r_1} \right)$$

$$T_2 - T_1 = C_2 \left(\frac{r_1 - r_2}{r_2 r_1} \right) \quad \text{HENCE} \quad C_2$$

TEMPERATURE PROFILE

$$\begin{aligned}T &= C_1 + \frac{C_2}{r} \\&= T_1 - \frac{C_2}{r_1} + \frac{C_2}{r} \\&= T_1 - (T_2 - T_1) / \left(\frac{r_1 - r_2}{r_2 r_1} \right) r_1 \\&\quad + (T_2 - T_1) / \left(\frac{r_1 - r_2}{r_2 r_1} \right) r \\&= T_1 - (T_2 - T_1) / \left[\frac{r_1 - r_2}{r_2 r_1} (r_1 - r) \right] \\&= T_1 - (T_2 - T_1) \left[\frac{r_2 r_1}{(r_2 - r_1)(r - r_1)} \right]\end{aligned}$$

HEAT CONDUCTED

$$\begin{aligned}Q_r &= -k A_r \frac{dT}{dr} \quad \text{AREA VARIES WITH } r \\&= -k 4\pi r^2 \left(\frac{C_1}{r} - \frac{T}{r} \right) \\&= -k 4\pi r (C_1 - T) \\&= -k 4\pi r \left(-\frac{C_2}{r} \right) \\&= -k 4\pi (-C_2) \\&= -k 4\pi \left(-\frac{T_2 - T_1}{(r_1 - r_2)(r_2 r_1)} \right) \\&= -k 4\pi \left(\frac{r_2 r_1}{r_2 - r_1} \right) (T_2 - T_1)\end{aligned}$$

Table 7
Alloying elements in Zircaloy-2 and -4*

Element	Percent by Weight	
	Zr-2	Zr-4
Tin	1.45	1.45
Iron	0.14	0.21
Chromium	0.10	0.10
Nickel	0.05	nil.
Zirconium	Balance	Balance

*Nominal values from ASTM-B-353

Physical properties. Table 8 lists some important physical properties of Zircaloy-4. Its density is 412 lb/ft³ (0.24 lb/in.³). The coefficient of thermal expansion to 600F is $3.43 \times 10^{-6}/F$. Because of Zircaloy's relatively low thermal expansion and its low modulus of elasticity—approximately 11×10^6 psi at 600F—a given thermal gradient produces less thermal stress in Zircaloy than in other structural materials.

Table 8
Some physical properties of Zircaloy-4

Temp, F	Density lb/ft ³	Thermal Conduc- tivity Btu/sq ft, hr, F/in.	Coeffi- cient of Linear Expansion† (in./in. F) 10 ⁶	Specific Heat	Modulus of Elasticity in Tension (Static Value) psi/10 ⁶
70	412	98		0.070	14
200		101	3.24	0.073	13
400		107	3.33	0.077	12
600		113	3.43	0.081	11
800		120	3.53	0.085	9
1000		131	3.62	0.086	7
1200		142	3.72	0.086	6

†Between 70F and temperature shown.

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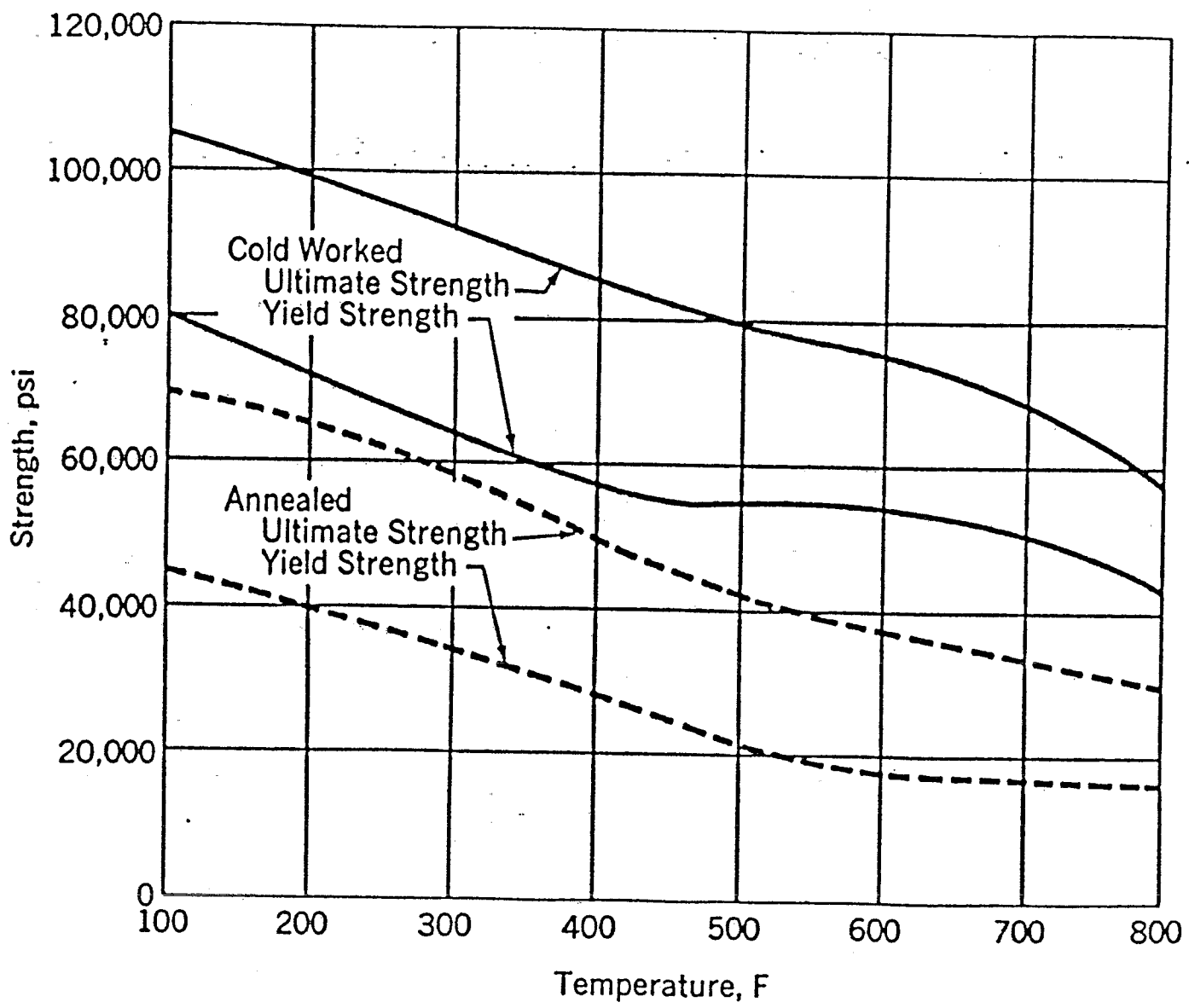
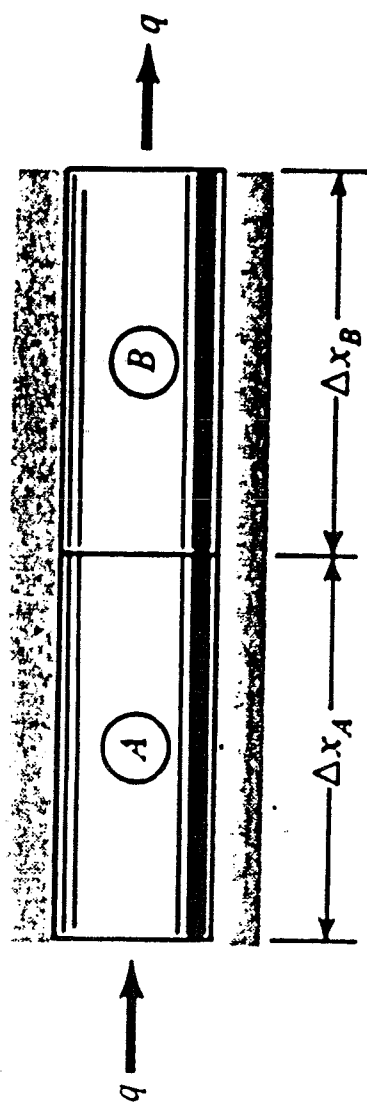
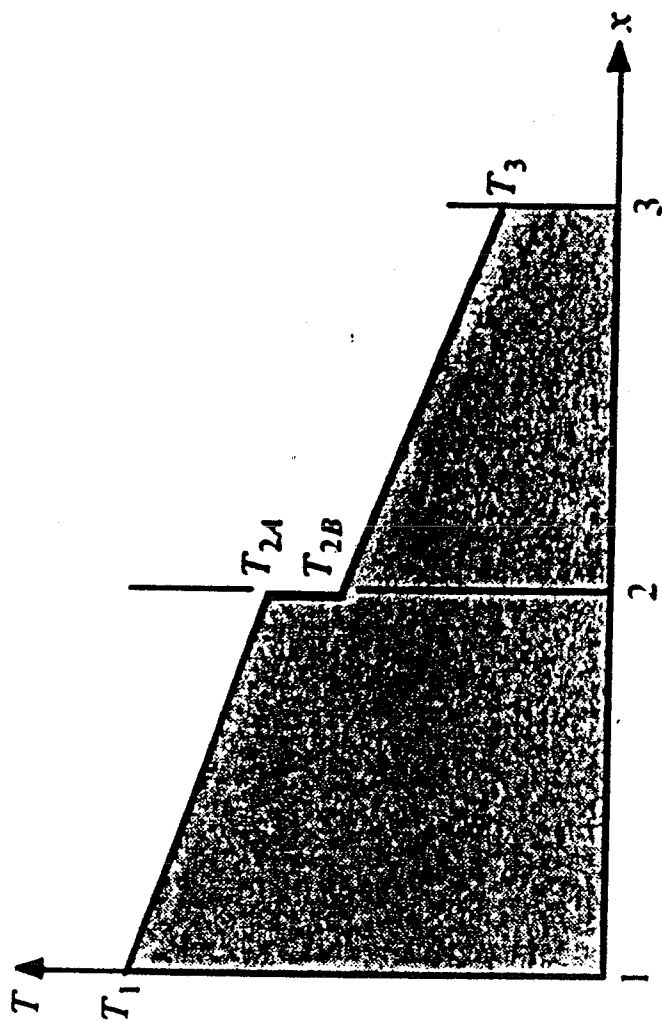


Fig. 14 Mechanical strength of Zircaloy-4 in the cold-worked and annealed conditions.



(a)



(b)

Fig. 2-14 Illustrations of thermal-contact-resistance effect: (a) physical situation; (b) temperature profile.

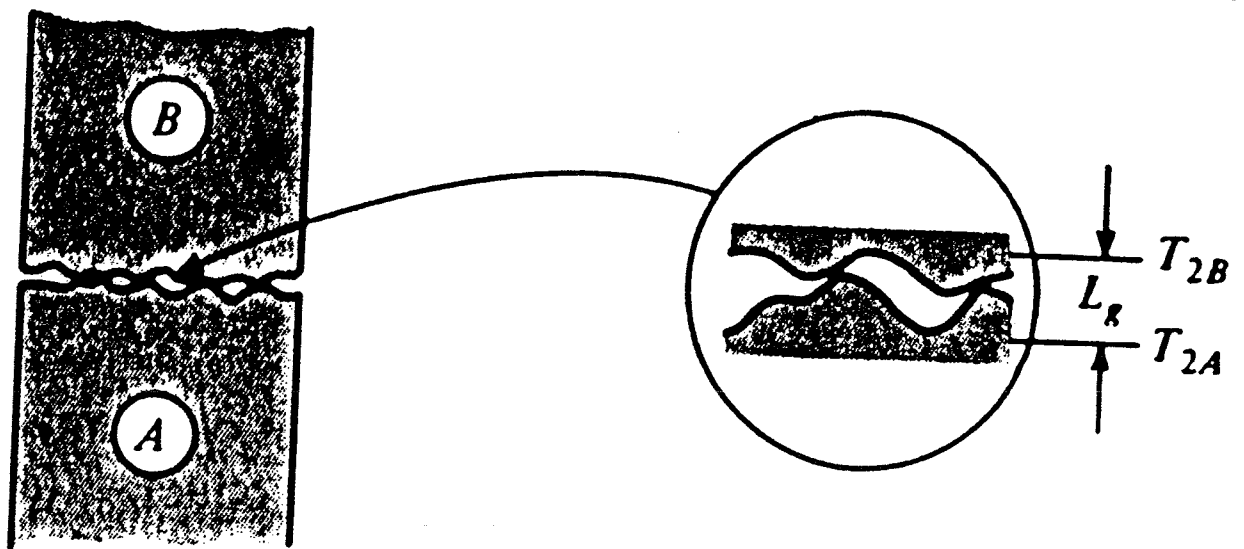
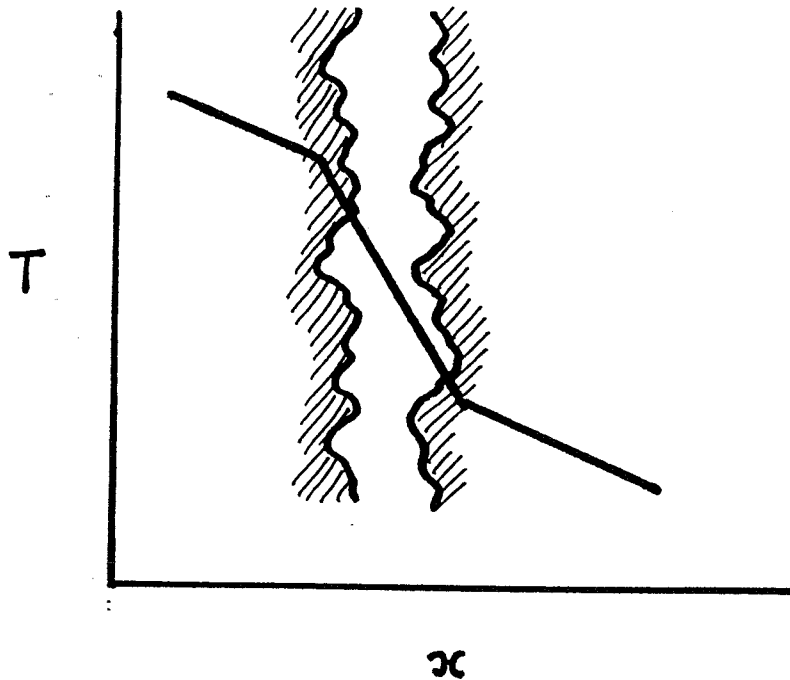


Fig. 2-15 Joint-roughness model for analysis of thermal contact resistance.

GAP CONDUCTANCE / RESISTANCE

TEMPERATURE PROFILE



TYPICAL RESISTANCE

PWR PELLET-CLADDING GAP

$$h_{\text{INTERFACE}} = 6000 \text{ J/S m}^2\text{ }^{\circ}\text{C}$$

(GLASSTONE + SESONSKE)

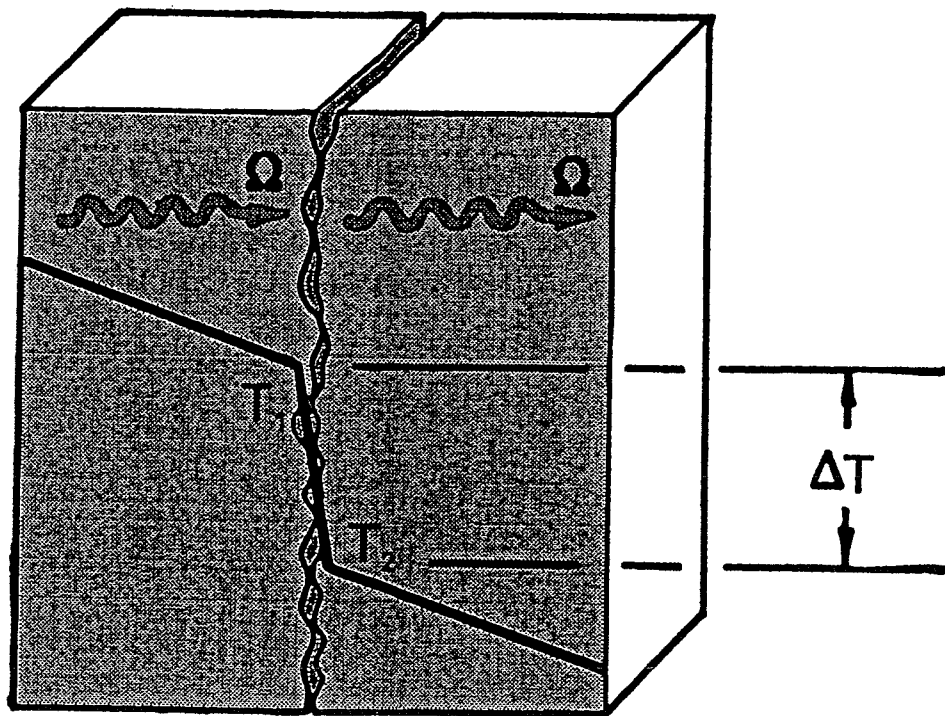


Figure 4 Contact resistance between two solids

$$t_2 - t_m = \frac{Qa^2}{2hb}$$

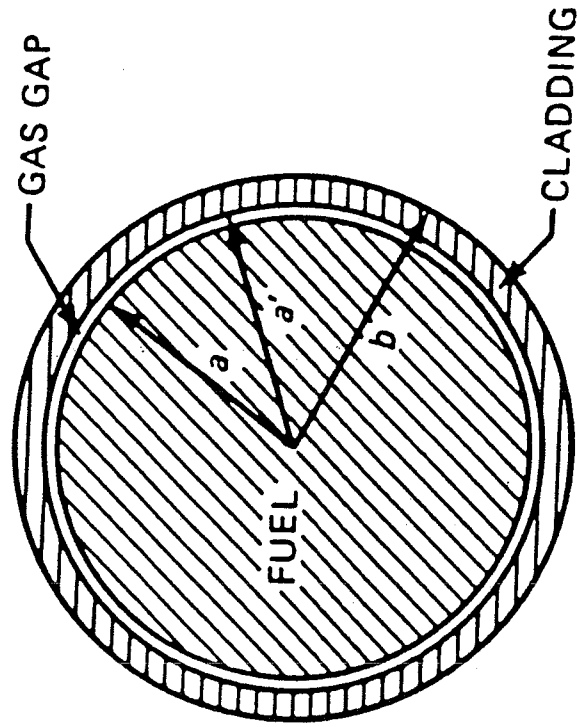


Fig. 6.6. Section through fuel pellet with gas gap and cladding.

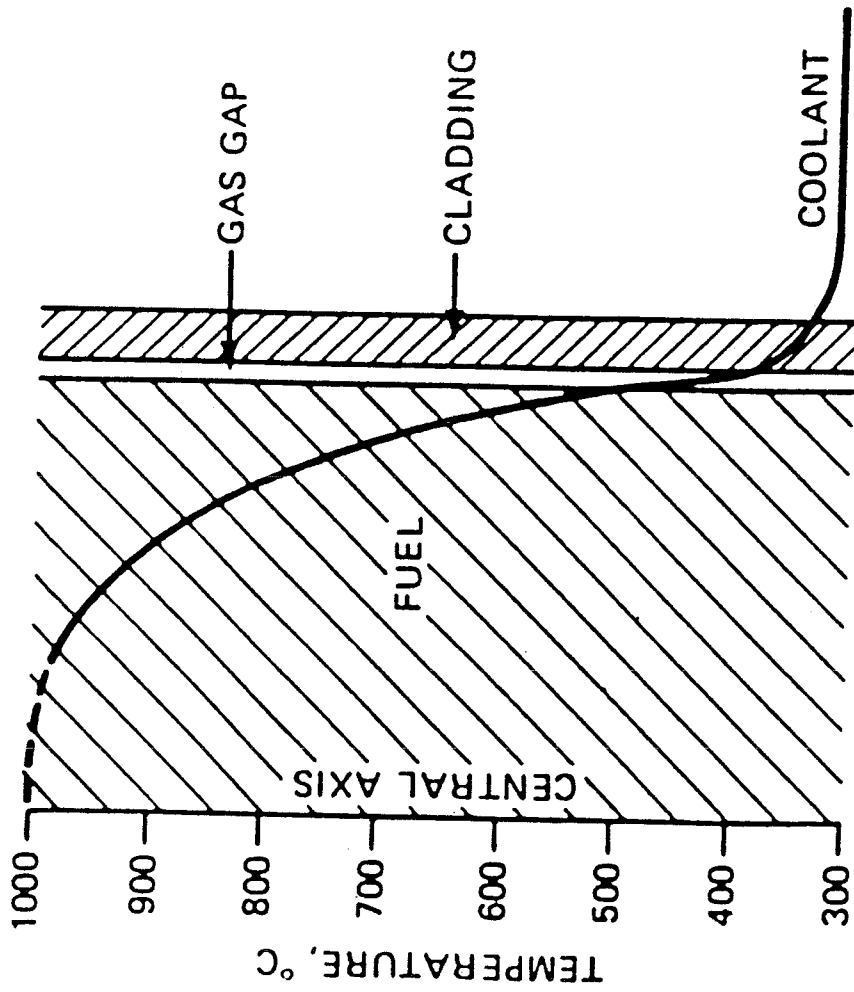


Fig. 6.7. Approximate radial temperature distribution in a fuel rod of a water-cooled reactor.

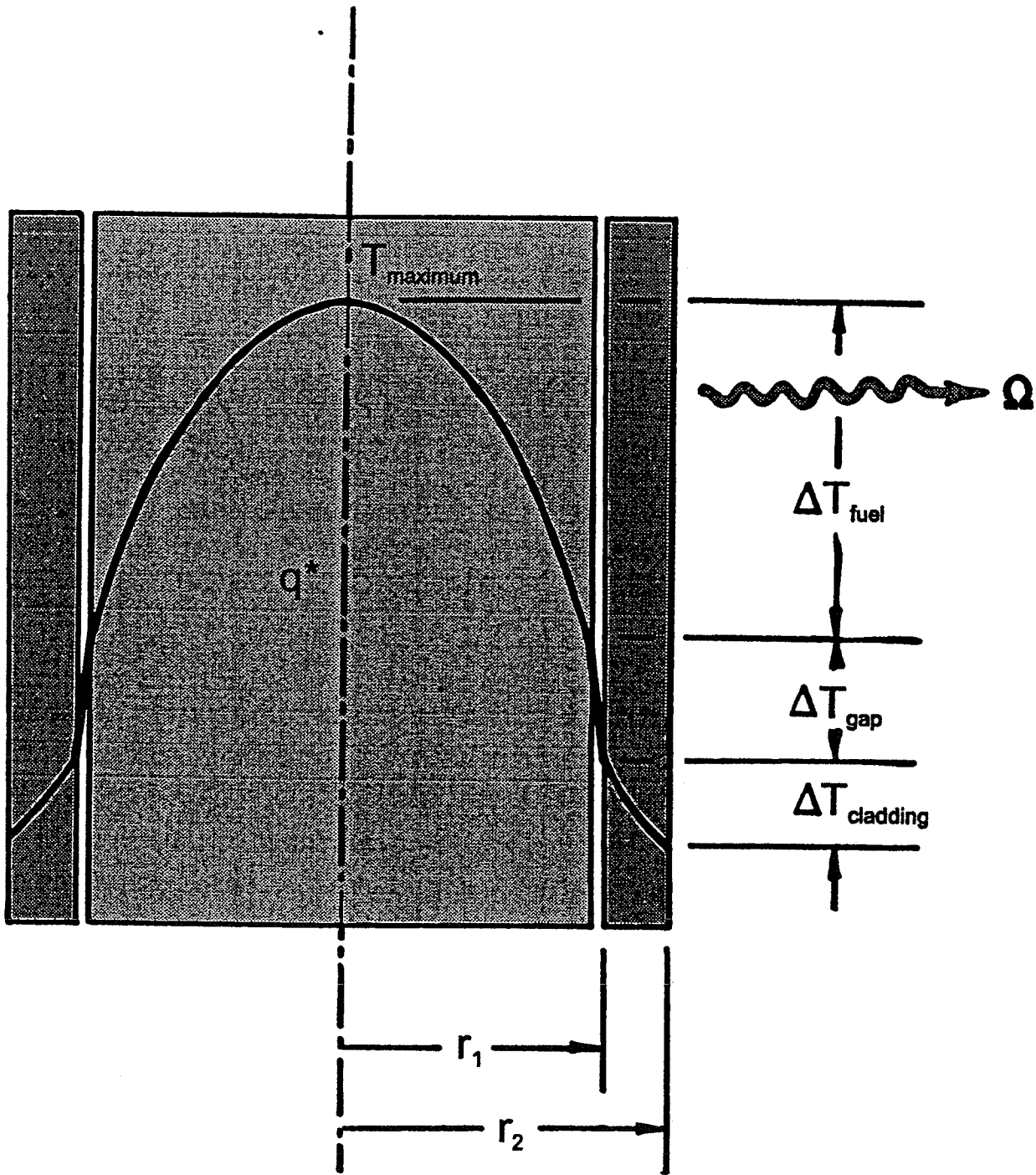


Figure 5 Heat conduction through a cylindrical fuel rod