CLADDING CONDUCTION

FLAT PLATE CONDUCTION

CONDUCTION EQUATION

$$Q_x = -kA\frac{dT}{dx}$$

GENERAL EQUATION

$$\frac{d^2T}{dx} = 0$$

$$\frac{dT}{dx} = c_1$$

$$T = c_1 x + c_2$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad x = x_1$$

$$T = T_2 \quad \text{AT} \quad x = x_2$$

THUS: $T_1 = c_1 x_1 + c_2$ HENCE c_2

$$T_2 = c_1 x_2 + c_2$$

$T_2 - T_1 = c_1(x_2 - x_1)$ HENCE c_1

TEMPERATURE PROFILE

$$T = c_1 x + c_2$$

$$= \left(\frac{T_2 - T_1}{x_2 - x_1}\right)x + T_1 - \left(\frac{T_2 - T_1}{x_2 - x_1}\right)x_1$$

$$= T_1 + \left(\frac{T_2 - T_1}{x_2 - x_1}\right)(x - x_1)$$

$$= T_1 + \left(\frac{x - x_1}{x_2 - x_1}\right)(T_2 - T_1)$$

HEAT CONDUCTED

$$Q_x = -kA\frac{dT}{dx}$$

$$= -kA\,c_1$$

$$= -kA\frac{T_2-T_1}{x_2-x_1}$$

$$\frac{Q_x}{A} = -k\left(\frac{T_2-T_1}{x_2-x_1}\right)$$

$$q_x = -k\left(\frac{T_2-T_1}{x_2-x_1}\right)$$

NOTE THAT AREA A IS CONSTANT FOR FLAT PLATE BUT NOT FOR CYLINDRICAL AND SPHERICAL SHELLS

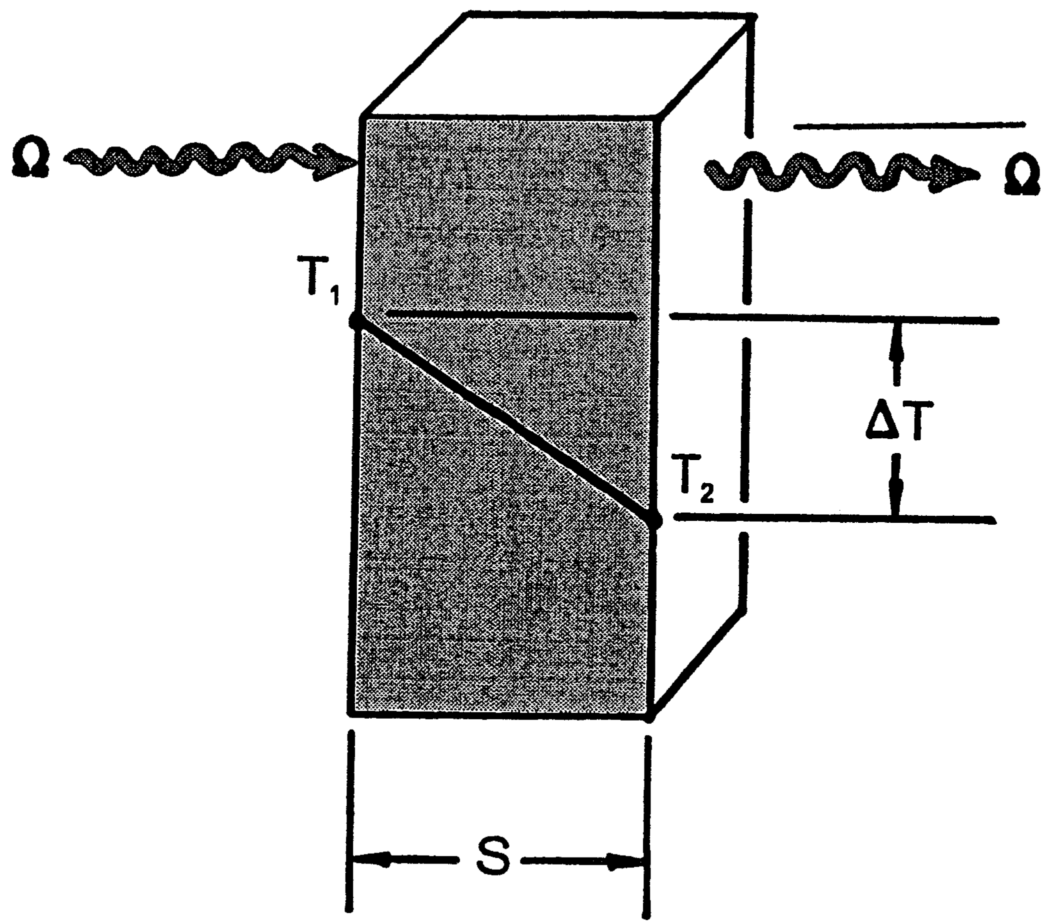

Figure 1 Heat conduction through a solid flat slab

CYLINDRICAL SHELL CONDUCTION

CONDUCTION EQUATION

$$Q_r = -kA_r \frac{dT}{dr}$$

GENERAL EQUATION

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

$$r\frac{d^2T}{dr^2} + \frac{dT}{dr} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$$

$$r\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r}$$

$$T = C_1 \ln r + C_2$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad r = r_1$$

$$T = T_2 \quad \text{AT} \quad r = r_2$$

THUS $T_1 = C_1 \ln r_1 + C_2$ HENCE C_2

$$T_2 = C_1 \ln r_2 + C_2$$

$$T_2 - T_1 = C_1 \ln r_2 - C_1 \ln r_1$$

$T_2 - T_1 = C_1 \ln r_2/r_1$ HENCE C_1

TEMPERATURE PROFILE

$$T = c_1 \ln r + c_2$$

$$= \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r + T_1 - \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r_1$$

$$= T_1 + \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right)(\ln r - \ln r_1)$$

$$= T_1 + \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) \ln r/r_1$$

HEAT CONDUCTED

$$Q_r = -k A_r \frac{dT}{dr} \qquad \text{AREA VARIES WITH } r$$

$$= -k\, 2\pi r L \frac{c_1}{r}$$

$$= -k\, 2\pi r L \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right) / r$$

$$= -k\, 2\pi L \left(\frac{T_2 - T_1}{\ln r_2/r_1}\right)$$

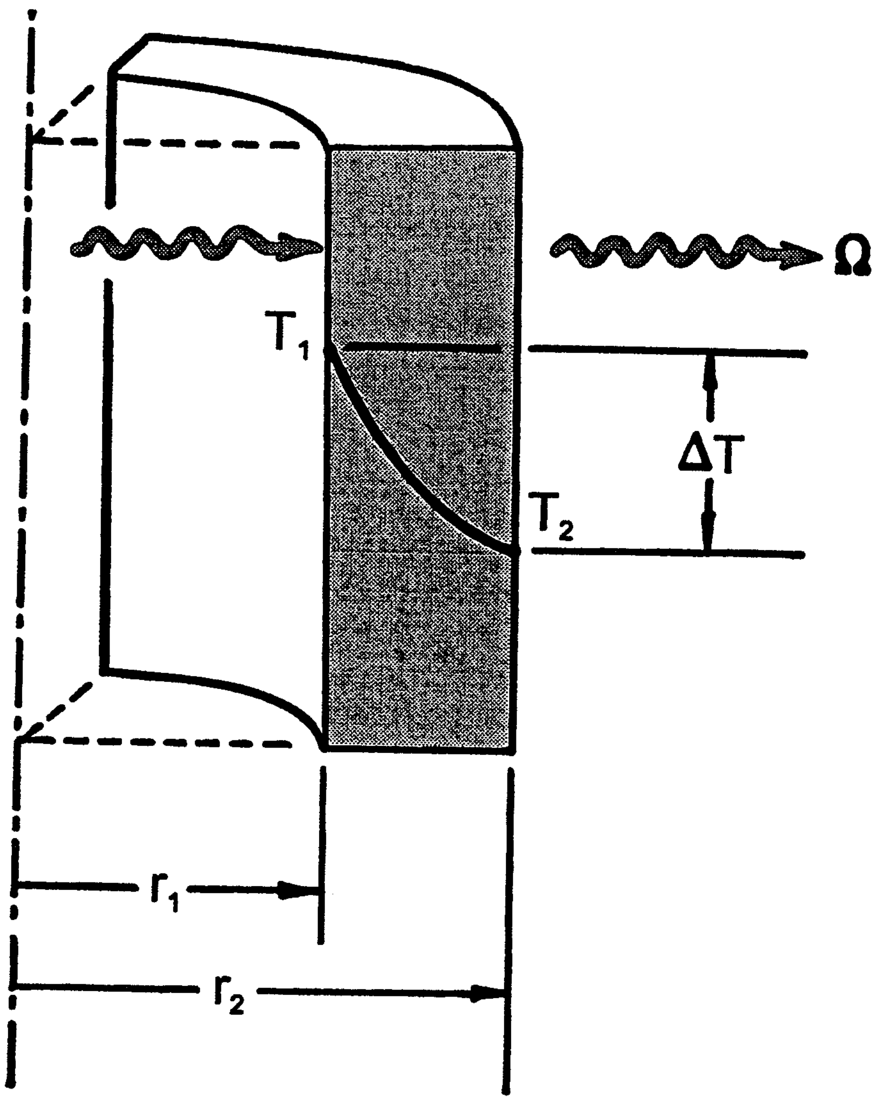

Figure 3 Heat conduction through a solid cylindrical wall

SPHERICAL SHELL CONDUCTION

CONDUCTION EQUATION

$$Q_r = -kA_r \frac{dT}{dr}$$

GENERAL EQUATION

$$\frac{d^2T}{dr^2} + \frac{2}{r}\frac{dT}{dr} = 0$$

$$r\frac{d^2T}{dr^2} + 2\frac{dT}{dr} = 0$$

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right) + \frac{dT}{dr} = 0$$

$$r\frac{dT}{dr} + T = C_1 \qquad \frac{dT}{dr} = \frac{C_1}{r} - \frac{T}{r}$$

$$Tr = C_1 r + C_2$$

$$T = C_1 + \frac{C_2}{r} \qquad C_1 - T = -\frac{C_2}{r}$$

SUBSTITUTE BOUNDARY CONDITIONS FOR CONSTANTS

$$T = T_1 \quad \text{AT} \quad r = r_1$$

$$T = T_2 \quad \text{AT} \quad r = r_2$$

THUS: $T_1 = C_1 + \frac{C_2}{r_1}$ HENCE C_1

$$T_2 = C_1 + \frac{C_2}{r_2}$$

$$T_2 - T_1 = \frac{C_2}{r_2} - \frac{C_2}{r_1}$$

$$T_2 - T_1 = C_2\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

$$T_2 - T_1 = C_2\left(\frac{r_1 - r_2}{r_2 r_1}\right)$$ HENCE C_2

TEMPERATURE PROFILE

$$T = c_1 + \frac{c_2}{r}$$

$$= T_1 - \frac{c_2}{r_1} + \frac{c_2}{r}$$

$$= T_1 - (T_2 - T_1) / \left(\frac{r_1 - r_2}{r_2 r_1} \right) r_1$$

$$+ (T_2 - T_1) / \left(\frac{r_1 - r_2}{r_2 r_1} \right) r$$

$$= T_1 - (T_2 - T_1) / \left[\frac{r_1 - r_2}{r_2 r_1} (r_1 - r) \right]$$

$$= T_1 - (T_2 - T_1) \left[\frac{r_2 r_1}{(r_2 - r_1)(r - r_1)} \right]$$

HEAT CONDUCTED

$$Q_r = -k A_r \frac{dT}{dr} \quad \text{AREA VARIES WITH } r$$

$$= -k \, 4\pi r^2 \left(\frac{c_1}{r} - \frac{T}{r} \right)$$

$$= -k \, 4\pi r \, (c_1 - T)$$

$$= -k \, 4\pi r \left(-\frac{c_2}{r} \right)$$

$$= -k \, 4\pi \, (-c_2)$$

$$= -k \, 4\pi \left(-\frac{T_2 - T_1}{(r_1 - r_2)/(r_2 r_1)} \right)$$

$$= -k \, 4\pi \left(\frac{r_2 r_1}{r_2 - r_1} \right) (T_2 - T_1)$$

Table 7
Alloying elements in Zircaloy-2 and -4*

Element	Percent by Weight Zr-2	Zr-4
Tin	1.45	1.45
Iron	0.14	0.21
Chromium	0.10	0.10
Nickel	0.05	nil.
Zirconium	Balance	Balance

*Nominal values from ASTM-B-353

Physical properties. Table 8 lists some important physical properties of Zircaloy-4. Its density is 412 lb/ft^3 (0.24 lb/in.^{3}). The coefficient of thermal expansion to 600F is 3.43×10^{-6}/F. Because of Zircaloy's relatively low thermal expansion and its low modulus of elasticity—approximately 11×10^6 psi at 600F—a given thermal gradient produces less thermal stress in Zircaloy than in other structural materials.

Table 8
Some physical properties of Zircaloy-4

Temp, F	Density lb/ft^3	Thermal Conductivity Btu/sq ft, hr, F/in.	Coefficient of Linear Expansion† (in./in. F) 10^6	Specific Heat	Modulus of Elasticity in Tension (Static Value) psi/10^6
70	412	98		0.070	14
200		101	3.24	0.073	13
400		107	3.33	0.077	12
600		113	3.43	0.081	11
800		120	3.53	0.085	9
1000		131	3.62	0.086	7
1200		142	3.72	0.086	6

†Between 70F and temperature shown.

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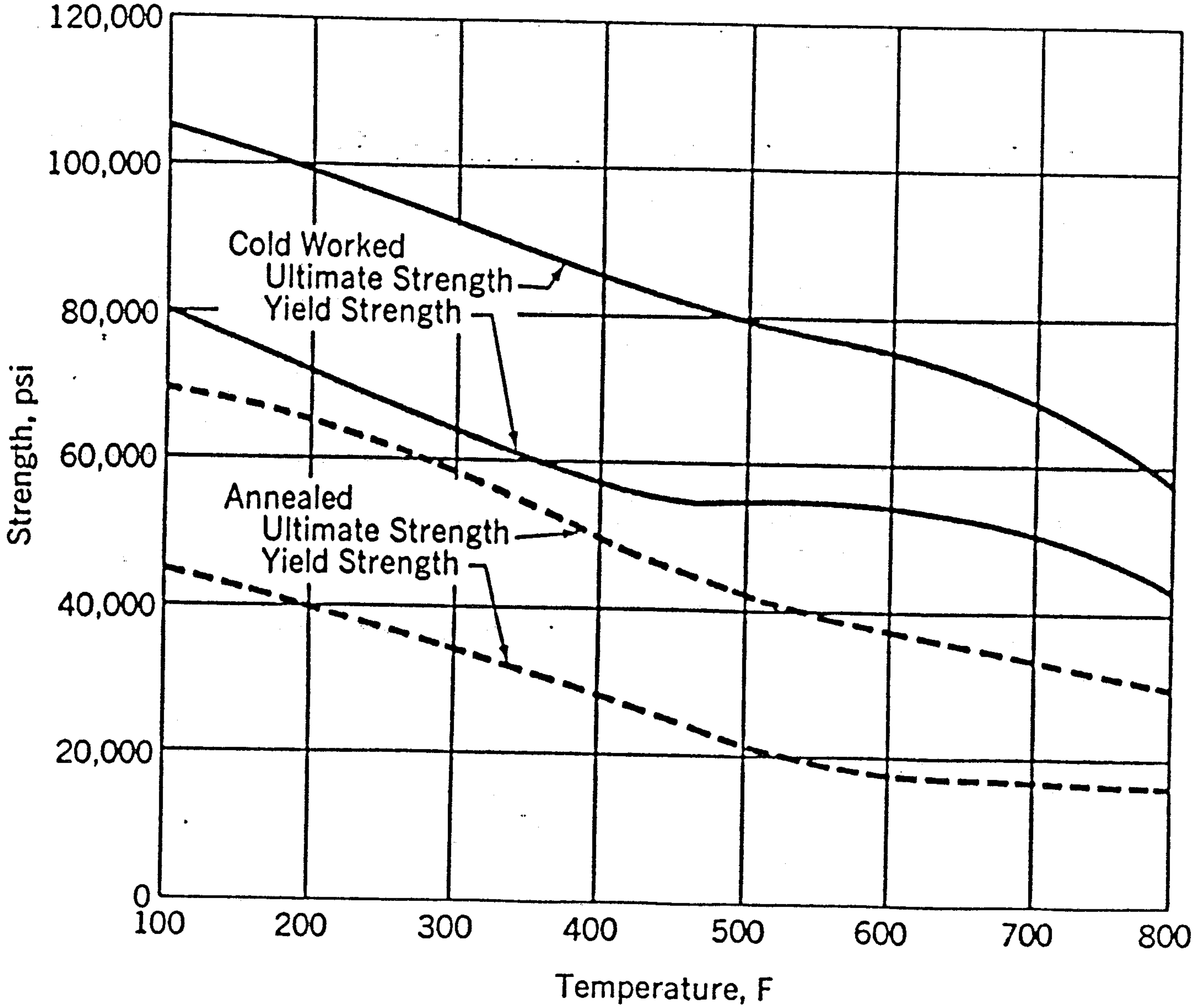


Fig. 14 Mechanical strength of Zircaloy-4 in the cold-worked and annealed conditions.

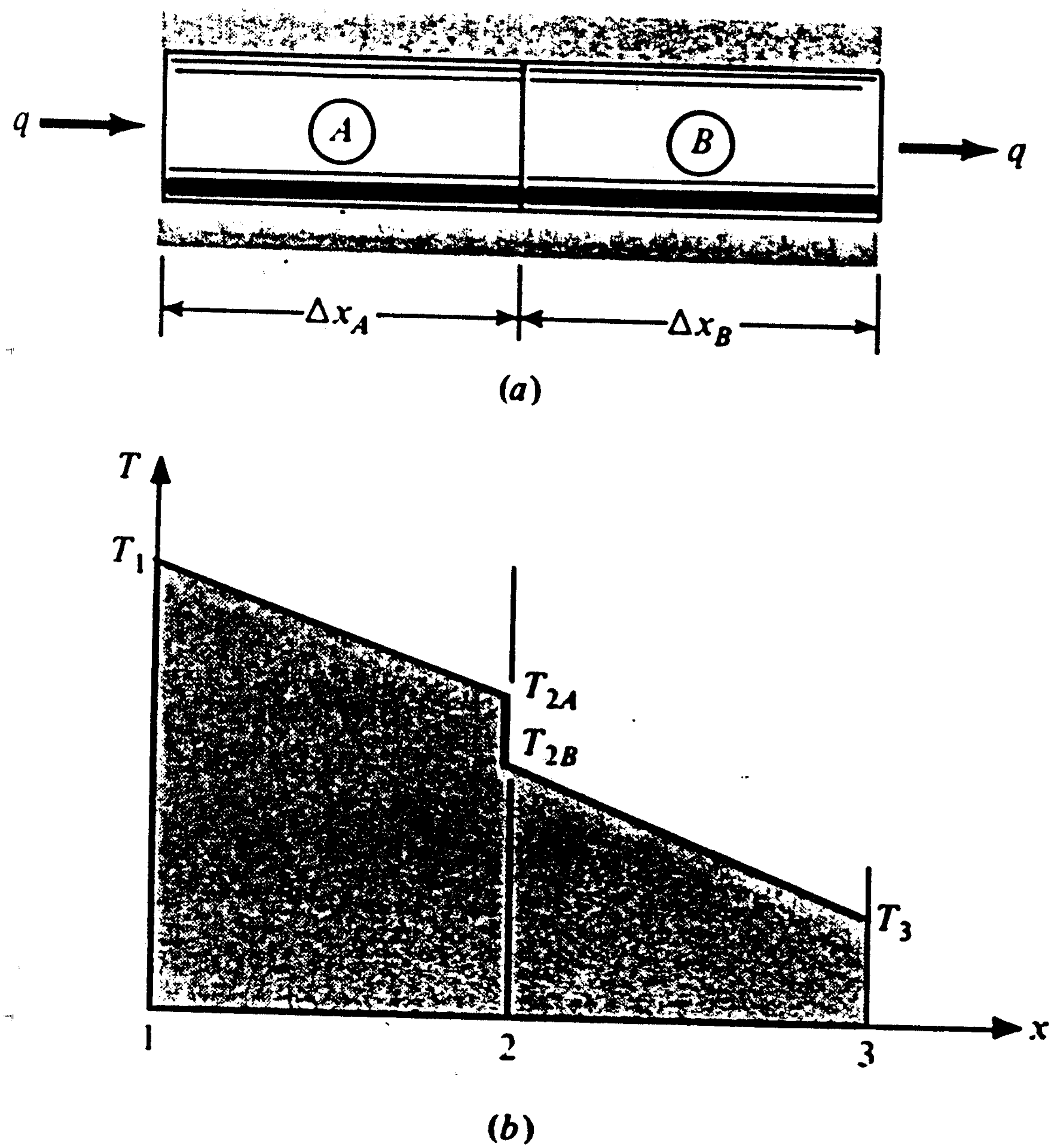


Fig. 2-14 Illustrations of thermal-contact-resistance effect: (*a*) physical situation; (*b*) temperature profile.

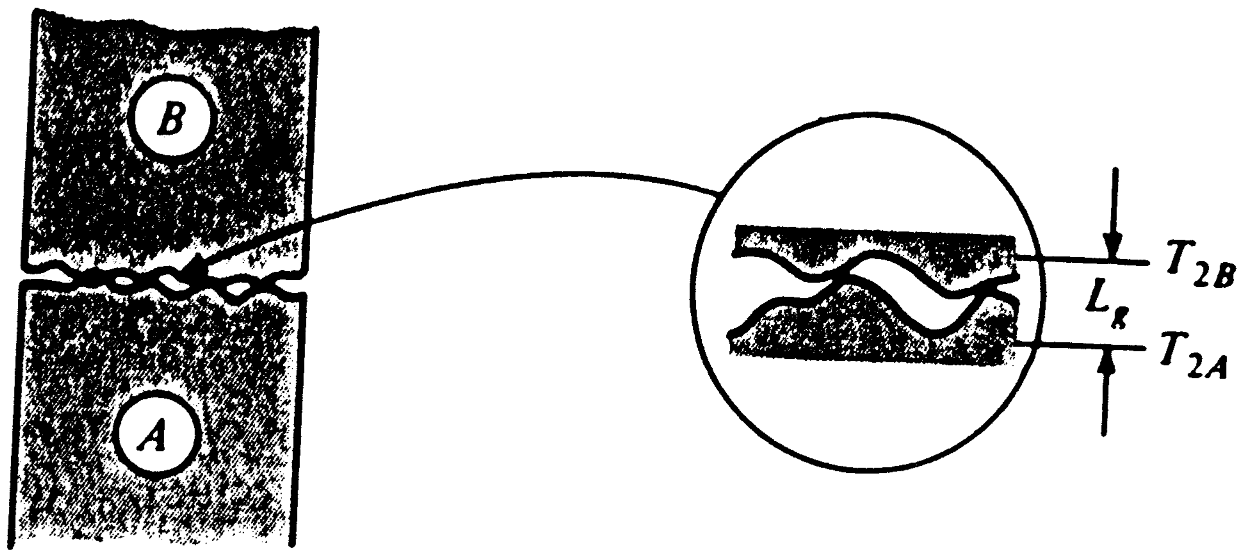
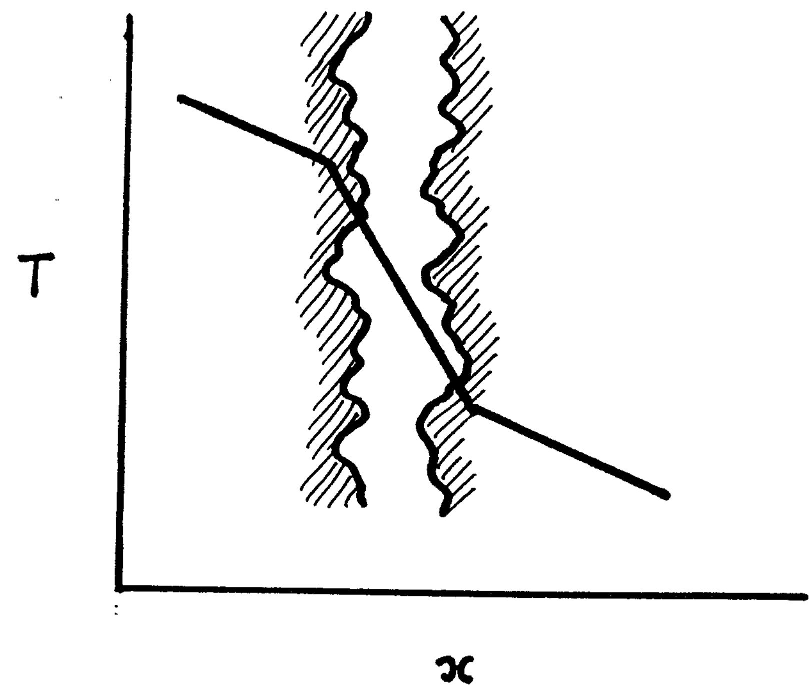


Fig. 2-15 Joint-roughness model for analysis of thermal contact resistance.

GAP CONDUCTANCE / RESISTANCE

TEMPERATURE PROFILE



TYPICAL RESISTANCE

PWR PELLET-CLADDING GAP

$h_{INTERFACE} = 6000 \ J/s\,m^2\,°C$

(GLASSTONE + SESONSKE)

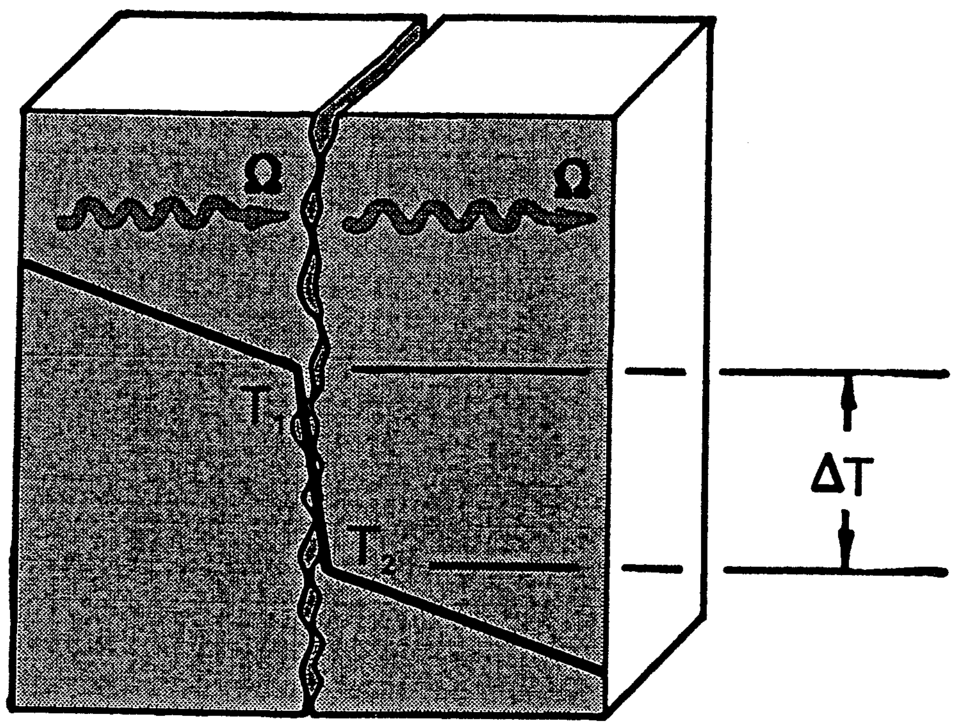


Figure 4 Contact resistance between two solids

$$t_2 - t_m = \frac{Qa^2}{2hb},$$

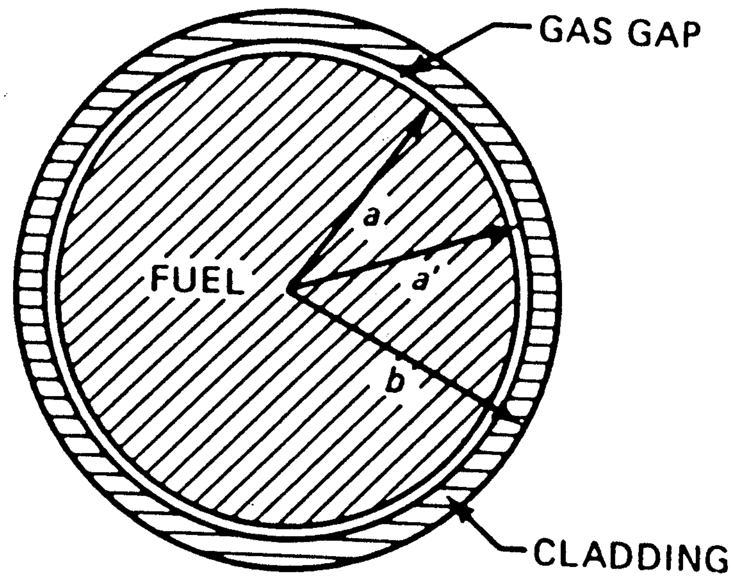


Fig. 6.6. Section through fuel pellet with gas gap and cladding.

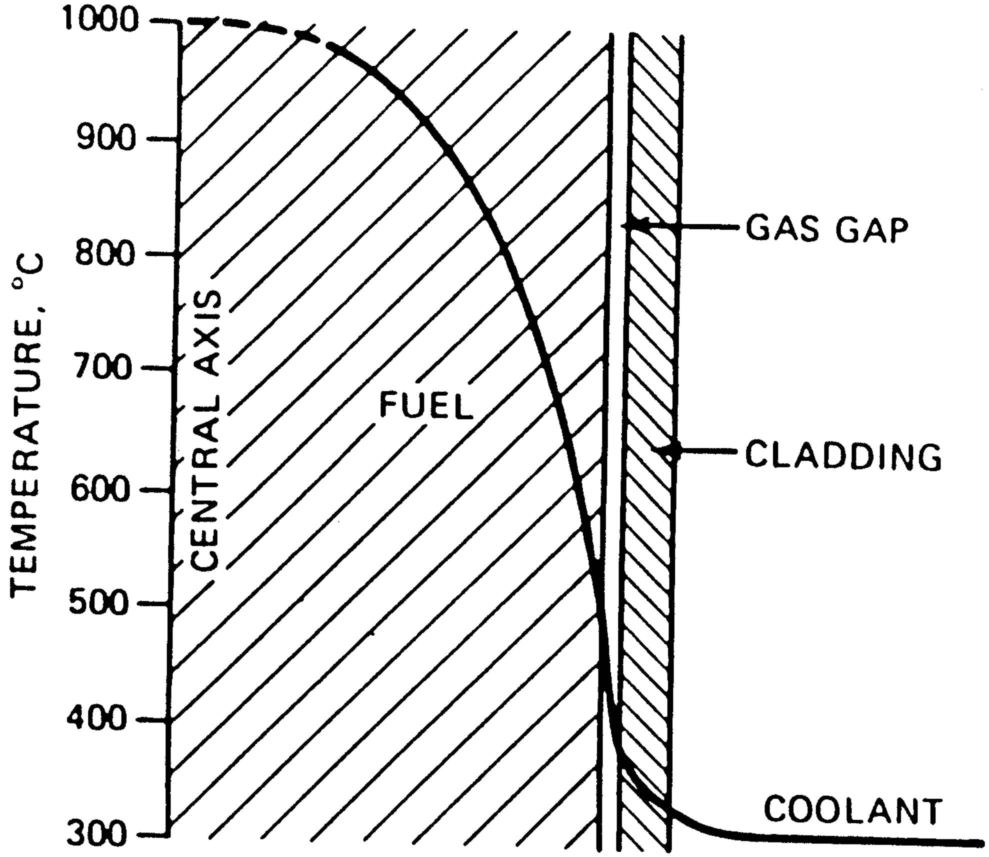


Fig. 6.7. Approximate radial temperature distribution in a fuel rod of a water-cooled reactor.

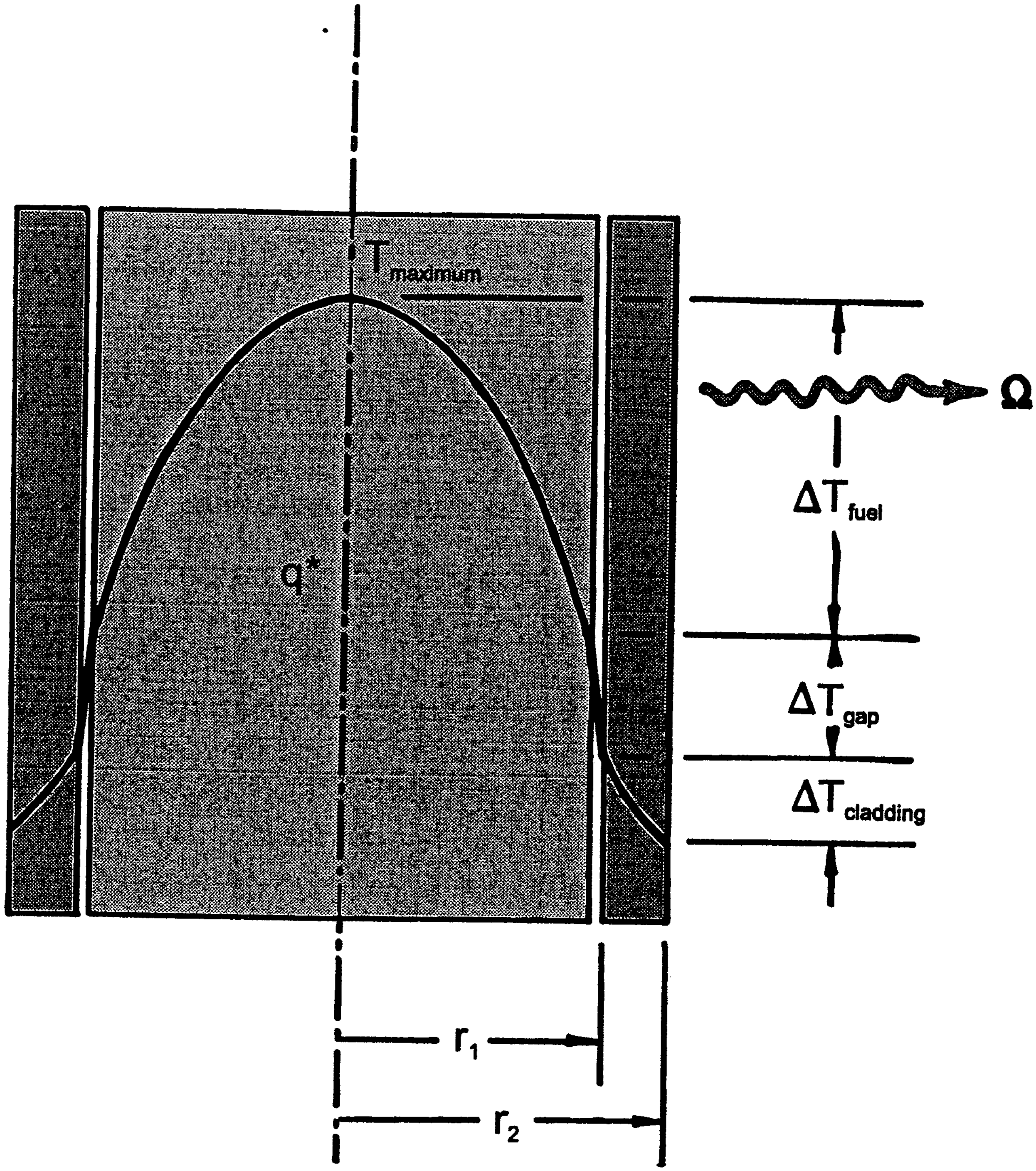


Figure 5 Heat conduction through a cylindrical fuel rod