900 MW PWR EXAMPLE

HEAT TRANSFER

UNIFORM HEAT GENERATION IN FUEL

$$t - t_1 = \frac{Q}{4k}(a^2 - r^2)$$

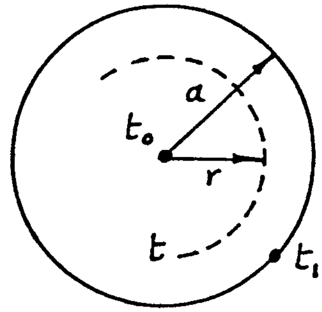
$$t_o - t_1 = \frac{Qa^2}{4k}$$

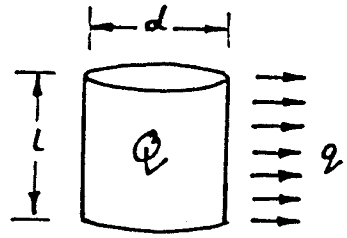
$$q/A = \frac{Q\,\pi/4\, d^2 l}{\pi d l}$$

$$q/A = \frac{Qd}{4} = \frac{Qa}{2}$$

$$t_o - t_1 = \frac{Qa^2}{4k} = \frac{Qa}{2}\,\frac{a}{2k}$$

$$t_o - t_1 = q/A\left(\frac{a}{2k}\right)$$





UNIFORM HEAT TRANSFER FROM FUEL

$$q = A_1 \Delta t \Big/ \left[\frac{r_1}{2k_f} + \frac{1}{h_g} + \frac{r_1 \ln(r_3/r_2)}{k_c} + \frac{r_1}{h_w r_3}\right]$$

$$\Delta t = q/A_1 \left[\frac{r_1}{2k_f} + \frac{1}{h_g} + \frac{r_1 \ln(r_3/r_2)}{k_c} + \frac{r_1}{h_w r_3}\right]$$

$$\Delta t = \frac{Q r_1}{2}\left[\frac{r_1}{2k_f} + \frac{r_1}{h_g r_1} + \frac{r_1 \ln(r_3/r_2)}{k_c} + \frac{r_1}{h_w r_3}\right]$$

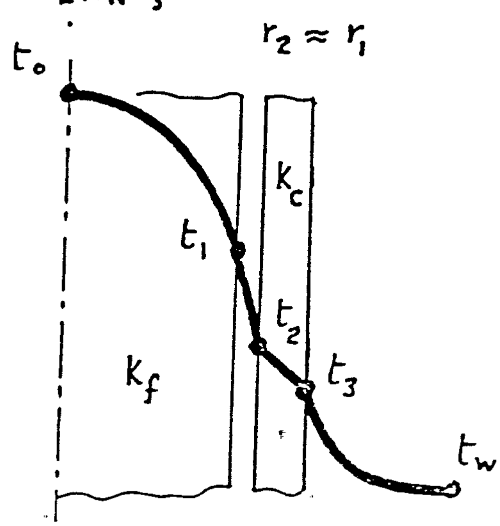
$$\Delta t = \frac{Q r_1^2}{4k_f} + \frac{Q r_1^2}{2 h_g r_1} + \frac{Q r_1^2 \ln(r_3/r_2)}{2k_c} + \frac{Q r_1^2}{2 h_w r_3}$$

$$t_o - t_1 = \frac{Q r_1^2}{4k_f}$$

$$t_1 - t_2 = \frac{Q r_1^2}{2 h_g r_1}$$

$$t_2 - t_3 = \frac{Q r_1^2}{2k_c}\ln\left(\frac{r_3}{r_2}\right)$$

$$t_3 - t_w = \frac{Q r_1^2}{2 h_w r_3}$$



FLUID FLOW

DIMENSIONLESS PARAMETERS

REYNOLD'S NUMBER : $R_e = \frac{VD}{\nu} = \frac{\rho V D}{\mu} \quad (\nu = \frac{\mu}{\rho})$

PRANDTL NUMBER : $P_r = \frac{C_p \mu}{k}$

NUSSELT NUMBER : $N_u = \frac{hD}{k}$

FLUID FRICTION IN CONDUIT

$$\Delta P = f \left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$ (IN CONSISTENT UNITS)

EQUIVALENT DIAMETER OF CHANNEL

$$D_e = 4 \frac{\text{CROSS SECTION}}{\text{WETTED PERIMETER}}$$

$$D_e = 4 \frac{\frac{1}{4}\pi D^2}{\pi D} = D$$ FOR CIRCULAR CHANNEL

CONVECTIVE HEAT TRANSFER INSIDE PIPE

$$N_u = 0.023 (R_e)^{0.8} (P_r)^{0.4}$$ FOR HEATING ($T_{PIPE} > T_{FLUID}$)

$$N_u = 0.023 (R_e)^{0.8} (P_r)^{0.33}$$

$$N_u = 0.023 (R_e)^{0.8} (P_r)^{0.3}$$ FOR COOLING ($T_{PIPE} < T_{FLUID}$)

CONVECTIVE HEAT TRANSFER ACROSS TUBE BANK

$$N_u = F_a \, 0.287 (R_e)^{0.6} (P_r)^{0.33}$$

F_a = ARRANGEMENT FACTOR

ALTERNATIVE FLUID FRICTION FORMULA

$$\Delta P = 4f \left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$ (IN CONSISTENT UNITS)

NUCLEAR FISSION ENERGY

CALCULATION OF POWER OUTPUT FOR TYPICAL PWR

DENSITY OF FUEL (UO_2)	ρ	=	$10.4\ g/cm^3$ (95% of theoretical)
AVERAGE CORE ENRICHMENT		=	2.8 %
URANIUM FISSION CROSS SECTION	σ_f	=	380 barns (effective)
AVOGADRO'S NUMBER	N_A	=	6.02×10^{23}
AVERAGE NEUTRON FLUX	ϕ	=	4.5×10^{13} neutrons/$cm^2 s$
ENERGY PER FISSION	E_f	=	32 pJ

HEAT RELEASE RATE

$$Q = \phi N_f \sigma_f E_f \qquad \frac{1}{m^2 s} \times \frac{1}{m^3} \times \frac{m^2}{1} \times \frac{J}{1} = \frac{J}{s\, m^3} = \frac{W}{m^3}$$

AVERAGE NEUTRON FLUX

$$\phi = 4.5 \times 10^{17} \text{ neutrons}/m^2 s \qquad \phi = nv$$

NUMBER OF URANIUM NUCLEI (TOTAL)

$$N = \text{Avogadro Number / Molecular Weight}$$

$$= \frac{6.02 \times 10^{23}}{270} \text{ nuclei/g}$$

$$= \frac{6.02 \times 10^{23}}{270 \times 10^{-3}} \text{ nuclei/kg}$$

$$= \frac{6.02 \times 10^{23}}{270 \times 10^{-3}} \rho \text{ nuclei}/m^3$$

$$= \frac{6.02 \times 10^{23} \times 10.4 \times 10^{3}}{270 \times 10^{-3}}$$

$$= 0.23 \times 10^{29} \text{ nuclei}/m^3$$

NUMBER OF FISSILE URANIUM NUCLEI (U-235)

$$N_f = 0.028 \times 0.23 \times 10^{29}$$
$$= 0.0064 \times 10^{29}$$
$$= 0.064 \times 10^{28} \text{ nuclei} / m^3$$

FISSION CROSS SECTION OF URANIUM (U-235)

$$\sigma_f = 380 \text{ barns}$$
$$= 380 \times 10^{-24} \text{ cm}^2$$
$$= 380 \times 10^{-28} \text{ m}^2$$

ENERGY RELEASED PER FISSION OF URANIUM (U-235)

$$E_f = 32 \times 10^{-12} \text{ J}$$

HEAT RELEASE RATE

$$Q = \phi N_f \sigma_f E_f$$

$$Q = 4.5 \times 10^{17} \times 0.064 \times 10^{28} \times 380 \times 10^{-28} \times 32 \times 10^{-12}$$
$$= 3500 \times 10^{5}$$
$$= 350 \times 10^{6} \text{ W} / m^3$$
$$= 350 \text{ MW} / m^3$$

FUEL ELEMENT GEOMETRY FOR TYPICAL PWR (KOEBERG)

NUMBER OF FUEL ELEMENTS IN CORE	=	157
NUMBER OF FUEL RODS PER ELEMENT	=	264
FUEL PELLET DIAMETER	=	8.19 mm
ACTIVE FUEL LENGTH	=	3.658 m

VOLUME OF FUEL IN CORE

$$V = n \frac{\pi}{4} d^2 l$$

$$V = 157 \times 264 \times \frac{\pi}{4} (8.19 \times 10^{-3})^2 \times 3.658$$
$$= 41448 \times 52.68 \times 10^{-6} \times 3.658$$
$$= 8.0 \times 10^{6} \times 10^{-6}$$
$$= 8.0 \; m^3$$

TOTAL ENERGY RELEASE FROM CORE

$$Q_T = QV$$
$$= 350 \times 8$$
$$= 2800 \; MW$$

THIS IS CONSISTANT WITH SPECIFIED VALUE OF 2775 MW

FUEL ELEMENT HEAT TRANSFER

CALCULATION OF HEAT FLUX

CONSIDER A TYPICAL PWR (KOEBERG)

FUEL PELLET DIAMETER	d	=	8.19 mm
FUEL PELLET CONDUCTIVITY	k	=	2.80 W/m °C

GENERAL EQUATION FOR HEAT FLOW OUT OF CYLINDER

$$\Delta t = \frac{Q a^2}{4k}$$

HEAT GENERATED IN FUEL

$$Q = 350 \times 10^6 \ W/m^3 \quad \text{(FROM ABOVE)}$$

TEMPERATURE DIFFERENCE BETWEEN CENTRE AND SURFACE

$$\Delta t = \frac{350 \times 10^6 \times (0.00819/2)^2}{4 \times 2.80}$$

$$= 524 \ ^\circ C$$

HEAT FLUX AT SURFACE OF FUEL PELLET

$$q/A = \text{HEAT GENERATED IN PELLET / SURFACE AREA OF PELLET}$$

$$= \frac{Q \frac{\pi}{4} d^2 l}{\pi d l} \quad \text{(RADIAL HEAT FLOW ONLY)}$$

$$= Q \frac{d}{4} \quad \left(Q \frac{a}{2} \right)$$

$$= 350 \times 10^6 \times \frac{0.00819}{4}$$

$$= 717\ 000 \ W/m^2$$

HEAT FLUX AT SURFACE OF FUEL ROD

$$q/F = 717\,000 \times \frac{8.19}{9.50}$$

$$= 618\,000 \quad W/m^2$$

$$= 62 \quad W/cm^2$$

THIS IS CONSISTANT WITH SPECIFIED VALUE OF 60 W/cm^2

CALCULATION OF TEMPERATURE PROFILE

CONSIDER A TYPICAL PWR

FUEL PELLET DIAMETER	d	=	8.19 mm
FUEL ROD DIAMETER	D	=	9.5 mm
FUEL CLADDING THICKNESS	t	=	0.57 mm
COOLANT TEMPERATURE	t_w	=	305 °C
HEAT TRANSFER COEFFICIENT	h_w	=	33 000 $W/m^2\,°C$
THERMAL CONDUCTIVITY	k_c	=	13 $W/m\,°C$
THERMAL CONDUCTIVITY	k_f	=	2.8 $W/m\,°C$
HEAT TRANSFER COEFFICIENT	h_g	=	6 000 $W/m^2\,°C$

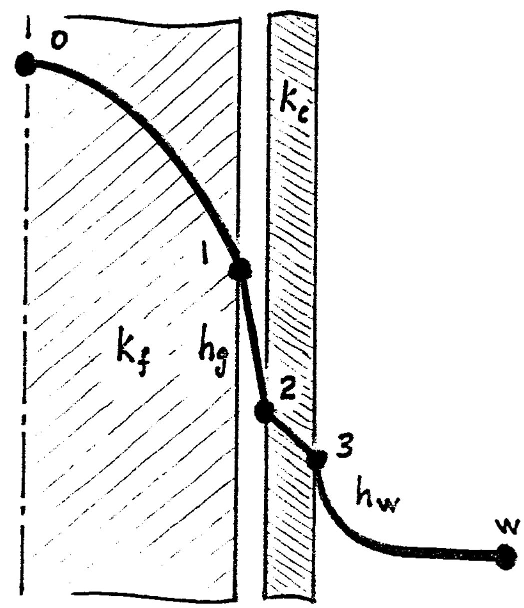
GENERAL EQUATIONS

$$t_0 - t_1 = \frac{Q\, r_1^2}{4k_f}$$

$$t_1 - t_2 = \frac{Q\, r_1^2}{2h_g r_1}$$

$$t_2 - t_3 = \frac{Q\, r_1^2}{2k_c} \ln\left(\frac{r_3}{r_2}\right)$$

$$t_3 - t_w = \frac{Q\, r_1^2}{2h_w r_3}$$



$r_1 = 0.00819/2 = 0.004095 \text{ m}$

$r_2 = 0.00475 - 0.00057 = 0.00418 \text{ m}$

$r_3 = 0.00950/2 = 0.00475 \text{ m}$

NOTE THAT EQUATIONS CAN BE WRITTEN AS FOLLOWS :

$$t_o - t_w = Q\left[\frac{r_1^2}{4k_f} + \frac{r_1^2}{2h_g r_1} + \frac{r_1^2}{2k_c}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_1^2}{2h_w r_3}\right]$$

$$\Delta t = Q\frac{r_1}{2}\left[\frac{r_1}{2k_f} + \frac{r_1}{h_g r_1} + \frac{r_1}{k_c}\ln\left(\frac{r_3}{r_2}\right) + \frac{r_1}{h_w r_3}\right]$$

$$\Delta t = q/A_1\left[\frac{r_1}{2k_f} + \frac{1}{h_g} + \frac{r_1 \ln(r_3/r_2)}{k_c} + \frac{r_1}{h_w r_3}\right]$$

$$q = A_1 \Delta t \Big/ \left[\frac{r_1}{2k_f} + \frac{1}{h_g} + \frac{r_1 \ln(r_3/r_2)}{k_c} + \frac{r_1}{h_w r_3}\right]$$

CALCULATION OF TEMPERATURES

$$t_3 - t_w = \frac{Q r_1^2}{2h_w r_3}$$

$$= \frac{350 \times 10^6 \times (0.004095)^2}{2 \times 33000 \times 0.00475}$$

$$= 19\ ^\circ C$$

$$t_2 - t_3 = \frac{Q r_1}{2k_c}\ln\left(\frac{r_3}{r_2}\right)$$

$$= \frac{350 \times 10^6 \times (0.004095)^2}{2 \times 13}\ln\left(\frac{0.00475}{0.00418}\right)$$

$$= 226 \ln(1.136)$$

$$= 29\ ^\circ C$$

$$t_1 - t_2 = \frac{Q r_1^2}{2h_g r_1}$$

$$= \frac{350 \times 10^6 \times (0.004095)^2}{2 \times 6000 \times 0.004095}$$

$$= 119\ ^\circ C$$

$$t_0 - t_1 = \frac{Q\, r_1^2}{4 k_f}$$

$$= \frac{350 \times 10^6 \times (0.004095)^2}{4 \times 2.8}$$

$$= 524\ ^\circ C$$

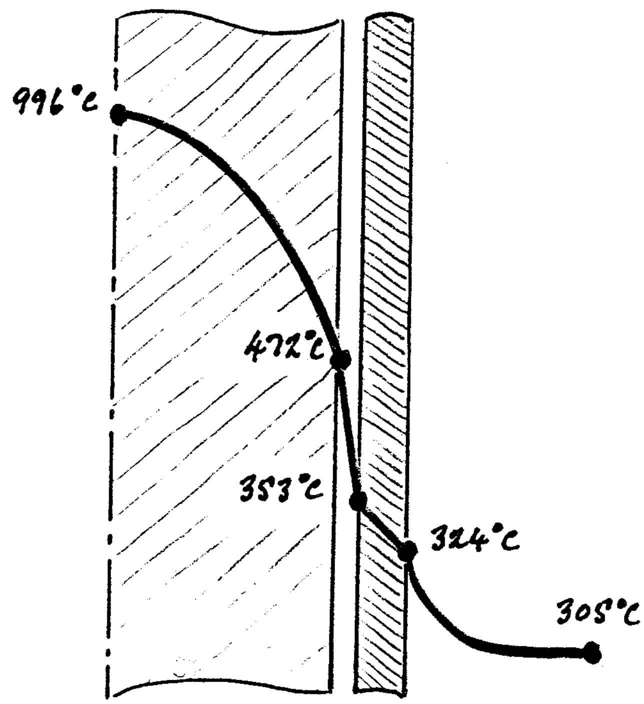
$$t_w = 305\ ^\circ C$$

$$t_3 = t_w + 19$$
$$= 305 + 19$$
$$= 324\ ^\circ C$$

$$t_2 = t_3 + 29$$
$$= 324 + 29$$
$$= 353\ ^\circ C$$

$$t_1 = t_2 + 119$$
$$= 353 + 119$$
$$= 472\ ^\circ C$$

$$t_0 = t_1 + 524$$
$$= 472 + 524$$
$$= 996\ ^\circ C$$



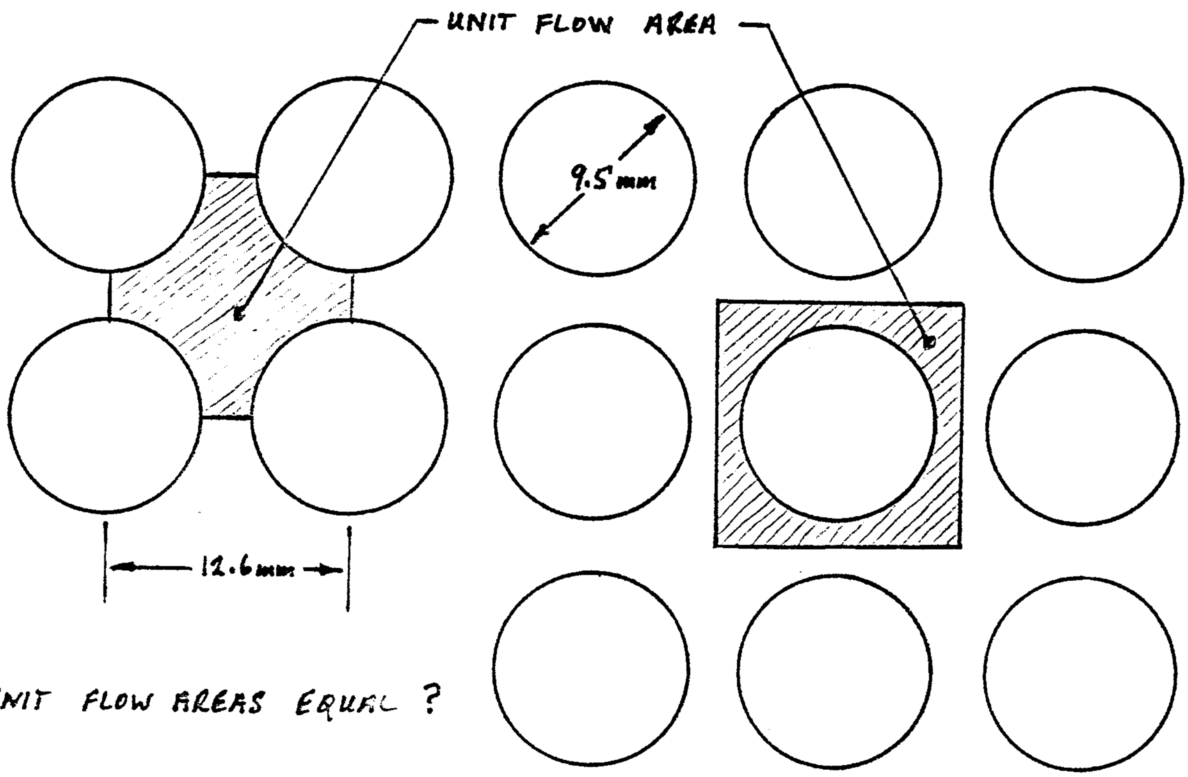
REACTOR HEAT REMOVAL

CALCULATION OF HYDRAULIC PARAMETERS

CONSIDER A TYPICAL PWR (KOEBERG)

FUEL ROD OUTER DIAMETER	D	=	9.5 mm
FUEL ROD PITCH	p	=	12.6 mm
FUEL ROD ARRAY		=	17 x 17 = 289
FUEL RODS PER ELEMENT		=	264
FUEL ELEMENTS IN CORE		=	157
COOLANT INLET TEMPERATURE		=	286 °C
COOLANT OUTLET TEMPERATURE		=	325 °C
COOLANT PRESSURE IN CORE		=	155 bar
FLOW RATE THROUG CORE		=	45 400 t/h
EFFECTIVE LENGTH OF FUEL ROD		=	3.658 m

HYDRAULIC CONDITIONS



ARE UNIT FLOW AREAS EQUAL ?

UNIT CROSS SECTION

$$A_u = p^2$$
$$= (0.0126)^2$$
$$= 0.0001588 \ m^2$$

ROD CROSS SECTION

$$A_R = \frac{\pi}{4} D^2$$
$$= \frac{\pi}{4} (0.0095)^2$$
$$= 0.0000709 \ m^2$$

UNIT FLOW AREA OR STREAM CROSS SECTION

$$A_s = A_u - A_R$$
$$= 0.0001588 - 0.0000709$$
$$= 0.0000879 \ m^2$$

STREAM WETTED PERIMETER

$$P_s = 4 \left(\tfrac{1}{4} \pi D \right) = \pi D$$
$$= \pi \ 0.0095$$
$$= 0.0298 \ m$$

HYDRAULIC DIAMETER

$$D_e = 4 \ (\text{STREAM CROSS SECTION} / \text{STREAM WETTED PERIMETER})$$
$$= 4 \ (A_s / P_s)$$
$$= 4 \ (0.0000879 / 0.0298)$$
$$= 0.01180 \ m$$

TOTAL FLOW RATE

$$W = 45\ 400\ 000 \ \ kg/hr$$
$$= 12\ 611 \ \ kg/s$$

UNIT FLOW RATE

NOTE WHICH NUMBER

$$\omega = 12\ 611 / (289 \times 157)$$
$$= 0.2779 \ \ kg/s$$

MASS FLOW RATE IN TERMS OF AREA

$$G = 0.2779 / 0.0000879$$
$$= 3162 \ \ kg/sm^2$$

CALCULATION OF HEAT TRANSFER COEFFICIENT

WATER CONDITIONS AND PARAMETERS IN PWR (KOEBERG)

AVERAGE TEMPERATURE	t	=	$305\ °C$	$(\frac{1}{2}(286 + 325))$
OPERATING PRESSURE	P	=	15 MPa	(155 bar)
SPECIFIC VOLUME	v	=	$1.399 \times 10^{-3}\ m^3/kg$	
DENSITY	ρ	=	$0.715 \times 10^{3}\ kg/m^3$	
THERMAL CONDUCTIVITY	k	=	$552 \times 10^{-3}\ W/m\,°C$	$(J/sm\,°C)$
DYNAMIC VISCOSITY	μ	=	$899 \times 10^{-7}\ kg/ms$	
KINEMATIC VISCOSITY	ν	=	$\mu/\rho = 1257 \times 10^{-10}\ m^2/s$	
SPECIFIC HEAT	c_p	=	$5.64\ kJ/kg\,°C$	

DIMENSIONLESS PARAMETERS

REYNOLDS NUMBER

$$Re = \frac{\rho V D}{\mu}$$

$$\rho V = G \qquad \frac{kg}{m^3} \times \frac{m}{s} = \frac{kg}{m^2 s}$$

$$D = D_e$$

$$Re = \frac{G D_e}{\mu}$$

$$= \frac{3162 \times 0.01180}{899 \times 10^{-7}} \qquad \frac{kg}{s\,m^2} \times \frac{m}{1} \times \frac{m\,s}{kg}$$

$$= 0.0415 \times 10^{7}$$

$$= 415\ 000$$

PRANDTL NUMBER

$$Pr = \frac{C_p \mu}{k}$$

$$= \frac{5.64 \times 899 \times 10^{-7}}{552 \times 10^{-5} \times 10^{-3}} \qquad \frac{kJ}{kg\,°C} \times \frac{kg}{m\,s} \times \frac{s\,m\,°C}{kJ}$$

$$= 9.71 \times 10^{-1}$$

$$= 0.971$$

NUSSELT NUMBER

$$Nu = \frac{h D}{k}$$

$$D = D_e$$

$$Nu = \frac{h D_e}{k}$$

$$= h \frac{0.01180}{552 \times 10^{-3}} \qquad \frac{W}{m^2\,°C} \times \frac{m}{1} \times \frac{m\,°C}{W}$$

$$= h\ 0.0000214 \times 10^{3}$$

$$= 0.0214\ h$$

CONVECTIVE HEAT TRANSFER FORMULA

$$N_u = 0.023\ (R_e)^{0.8}\ (P_r)^{0.4}$$

$$0.0214\ h = 0.023\ (415\ 000)^{0.8}\ (0.971)^{0.4}$$

$$h = 0.023 \times 31200 \times 0.988\ /\ 0.0214$$
$$= 33\ 000\ \ W/m^2\ °C$$

THIS IS THE VALUE ASSUMED IN TEMPERATURE PROFILE CALCULATION

CALCULATION OF COOLANT TEMPERATURE RISE

FUEL ROD HEAT FLUX FROM PREVIOUS CALCULATION

$$q/A = 62\ W/cm^2$$

THIS IS CONSISTANT WITH SPECIFIED VALUE OF 60 W/cm²

FUEL ROD SURFACE AREA

$$A = \pi D l$$

$$= \pi\ 0.0095 \times 3.658$$
$$= 0.109\ \ m^2$$

$$A_{TOTAL} = 0.109 \times 264 \times 157$$
$$= 4525\ \ m^2$$

TOTAL HEAT TRANSFERRED

$$q = 618\,000 \times 4525$$
$$= 2796\,000\,000 \quad W$$
$$= 2796 \quad MW$$

THIS IS CONSISTANT WITH SPECIFIED VALUE OF 2775 MW

COOLANT TEMPERATURE RISE

$$\Delta t = \frac{q}{c_p W}$$
$$= \frac{2796\,000}{5.64 \times 12\,611} \qquad \frac{kJ}{s} \times \frac{kg\,°C}{kJ} \times \frac{s}{kg}$$
$$= 39\,°C$$

SPECIFIED VALUE

$$\Delta t = 325 - 286$$
$$= 39\,°C$$

THUS TEMPERATURE RISE CALCULATION IS CONSISTANT

CALCULATION OF FRICTION DROP

GENERAL EQUATION

$$\Delta P = f\left(\frac{L}{D}\right)\left(\frac{V^2}{2g}\right)$$

IN CONSISTANT S.I. UNITS WITH $f = 4f$

$$\Delta P = 4f\left(\frac{L}{D_e}\right)\left(\frac{V^2}{2}\right)\rho \qquad \frac{m}{m} \times \frac{m^2}{s^2} \times \frac{kg}{m^3} = \frac{kg\,m}{s^2\,m^2} = \frac{N}{m^2} = Pa$$

$$V = \frac{G}{\rho} \qquad \frac{kg}{s\,m^2} \times \frac{m^3}{kg} = \frac{m}{s}$$

$$\Delta P = 4f\left(\frac{L}{D_e}\right)\left(\frac{G^2}{2\rho}\right)$$

$$= 4f\left(\frac{3.658}{0.01180}\right)\left(\frac{(3162)^2}{2 \times 0.715 \times 10^3}\right)$$

$$= 4f\ 310 \times 6992$$

$$= 8760\ 000\ f \quad N/m^2$$

$$Re = 415\ 000$$

$$= 4.15 \times 10^5$$

FROM GRAPH IN TEXT (FRICTION FACTOR VERSUS REYNOLDS NUMBER)

$$f = 3.4 \times 10^{-3} \quad \text{FOR SMOOTH PIPE}$$

$$\Delta P = 8760 \times 10^3 \times 3.4 \times 10^{-3}$$

$$= 29500 \quad Pa$$

$$= 30 \quad kPa$$

$$= 0.3 \quad bar$$