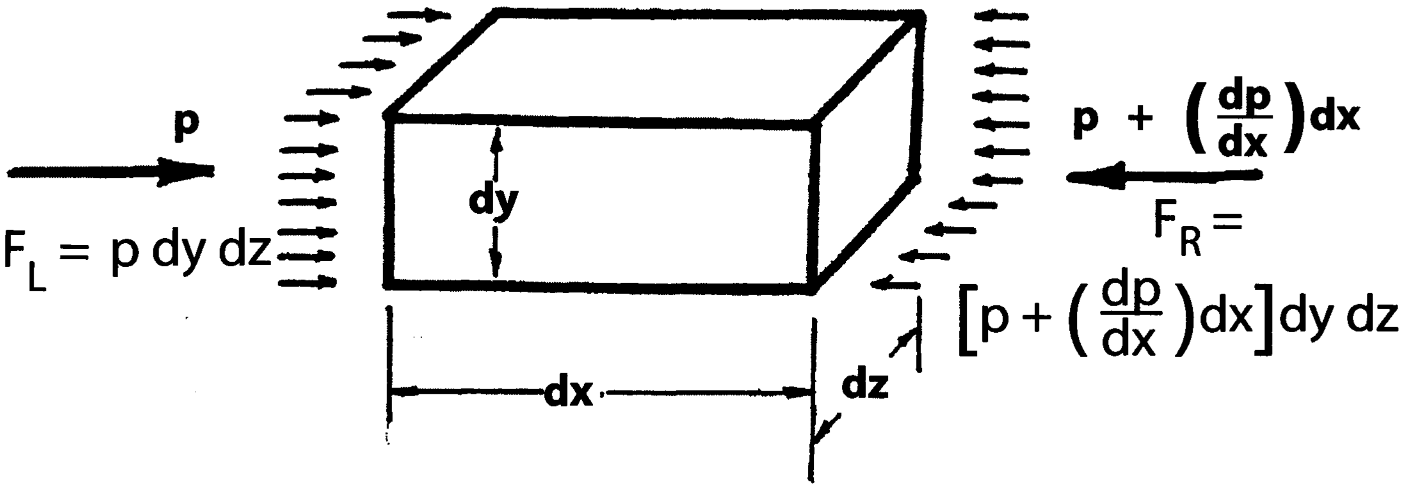
LECTURE 0

INTRODUCTION
MOMENTUM THEORY

EULER'S EQUATION

(FORCE = MASS X ACCELERATION)



Net Force $F = F_L - F_R$

$$= p\,dy\,dz - \left[p + \left(\frac{dp}{dx}\right)dx\right]dy\,dz$$

$$= -\frac{dp}{dx}(dx\,dy\,dz)$$

Fluid Mass $M = \rho(dx\,dy\,dz)$

Acceleration $a = \frac{dV}{dt}$

$$= \frac{dV}{dx}\frac{dx}{dt}$$

$$= \frac{dV}{dx}V$$

Net Force $F = m\,a$

$$-\frac{dp}{dx}(dx\,dy\,dz) = \rho(dx\,dy\,dz)\frac{dV}{dx}V$$

$$\boxed{dp = -\rho V dV}$$

MOMENTUM EQUATION

(CHANGE IN FLOW CREATES FORCE)

Euler's Equation

$$dp = -\rho V\, dV$$

Force equals Pressure x Area

$$A\, dp = -\rho VA\, dV$$

If $\rho VA = M$ is constant

$$A dp = -M\, dV$$

Integrating between ① and ②

$$A\int_1^2 dp = -M\int_1^2 dV$$

$$A(p_2 - p_1) = -M(V_2 - V_1)$$

If there is an external residual force acting in x-direction

$$A(p_2 - p_1) \neq -M(V_2 - V_1)$$

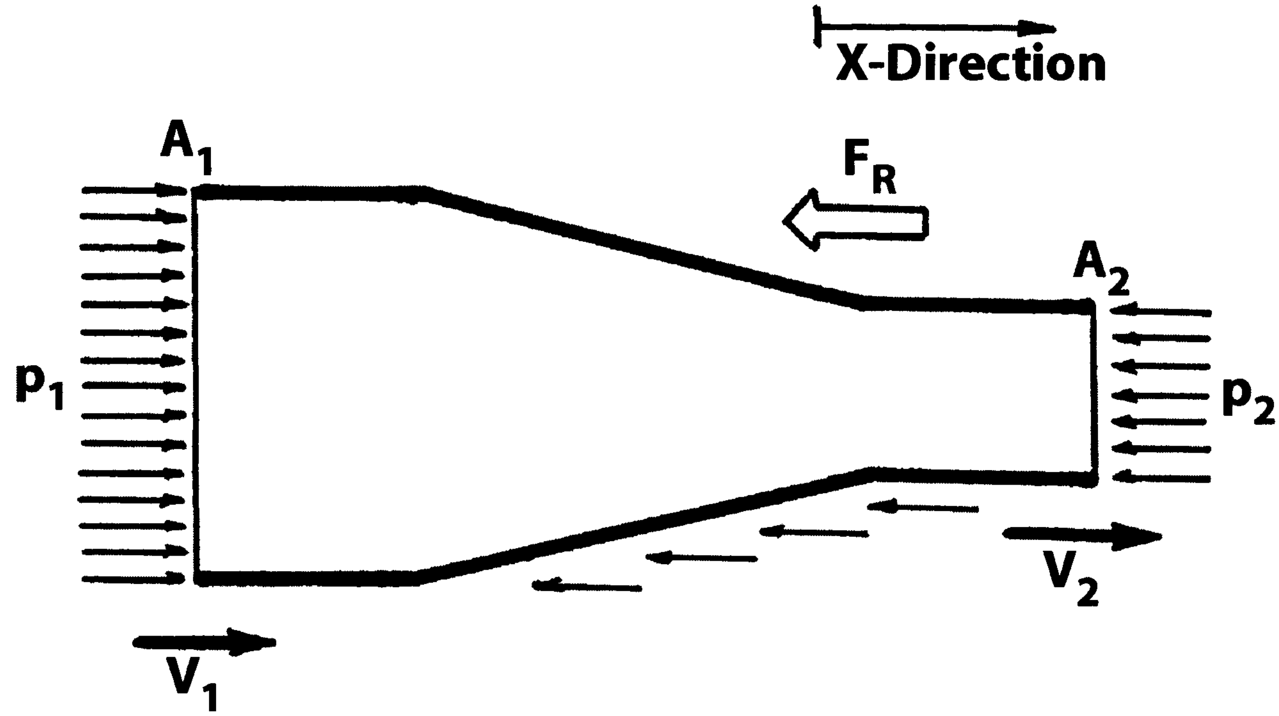
Force is the difference between the above terms

$$F = -A(p_2 - p_1) - M(V_2 - V_1)$$

$$\boxed{F = p_1 A - p_2 A - M(V_2 - V_1)}$$

APPLICATIONS

CONICAL PIPE



Forces in X - Direction (on fluid)

$$\Sigma F_x = p_1 A_1 - p_2 A_2 - F_R = \rho Q \Delta V_x$$

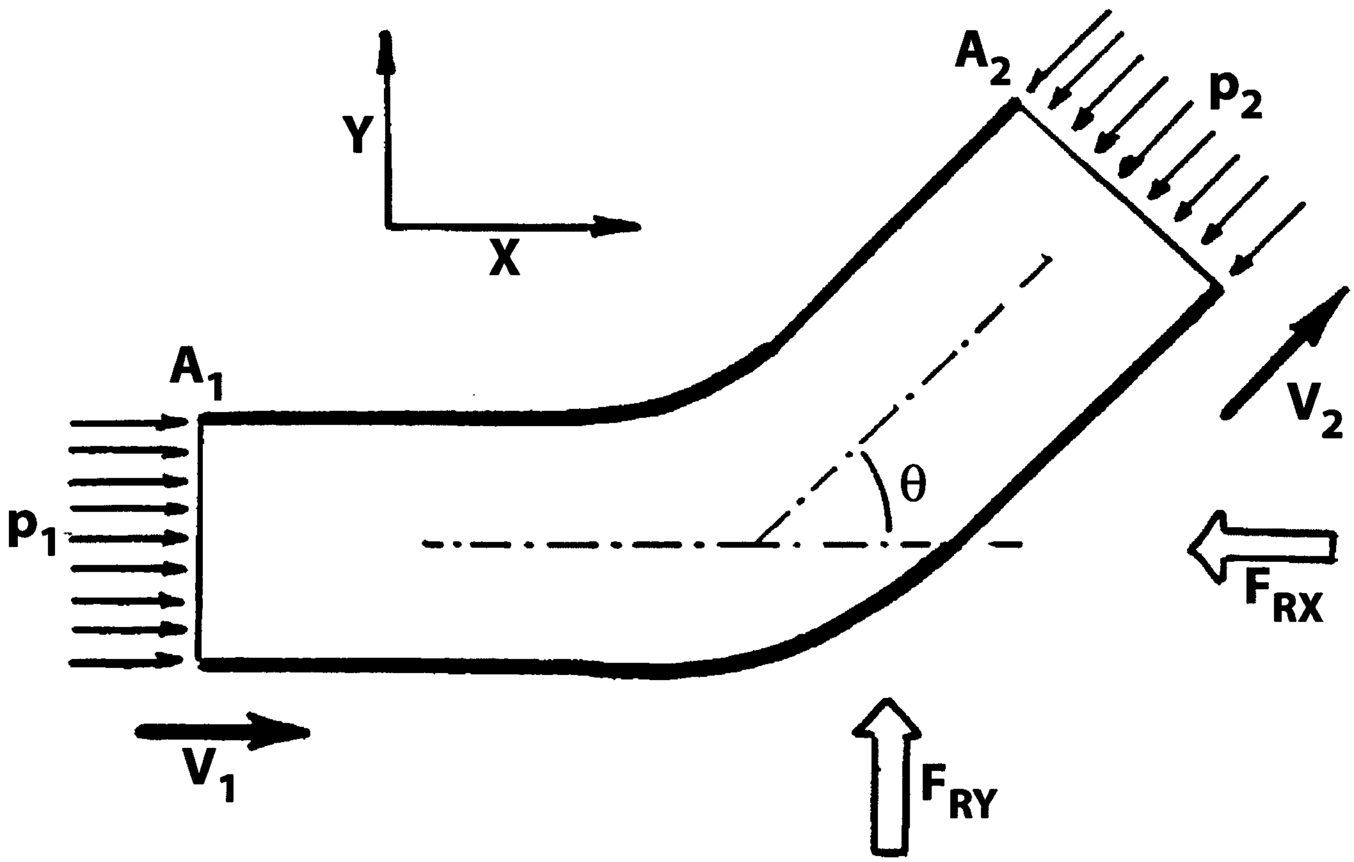
Reaction

* $$F_R = p_1 A_1 - p_2 A_2 - \rho Q (V_2 - V_1)$$

* Note sign convention

APPLICATIONS

ANGLED PIPE



Forces in X - Direction (on fluid)

$$\Sigma F_X = p_1A_1 - p_2A_2\cos\theta - F_{RX} = \rho Q \Delta V_X$$

Reaction

$$F_{RX} = p_1A_1 - p_2A_2\cos\theta - \rho Q(V_2\cos\theta - V_1)$$

Forces in Y - Direction (on fluid)

$$\Sigma F_Y = - p_2A_2\sin\theta + F_{RY} = \rho Q \Delta V_Y$$

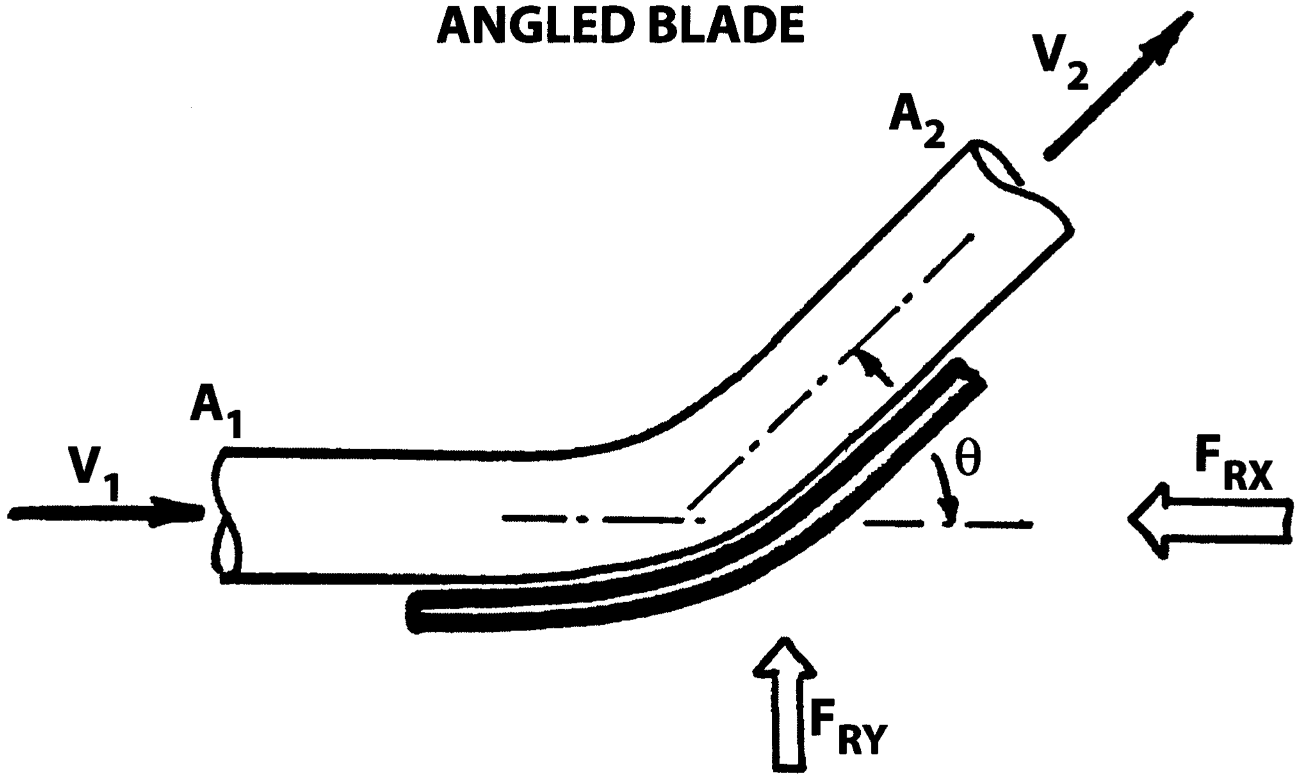
Reaction

$$F_{RY} = p_2A_2\sin\theta + \rho Q V_2\sin\theta$$

FORCES IN FREE JETS

MOMENTUM EQUATION APPLIED TO BLADES

ANGLED BLADE



Use Previous Equation for Flow in Pressure Conduit

Reaction in X - Direction

$$F_{RX} = p_1A_1 - p_2A_2\cos\theta - \rho Q(V_2\cos\theta - V_1)$$

Since p_1 and p_2 are zero

$$F_{RX} = -\rho Q(V_2\cos\theta - V_1)$$

Reaction in Y - Direction

$$F_{RY} = p_2A_2\sin\theta + \rho QV_2\sin\theta$$

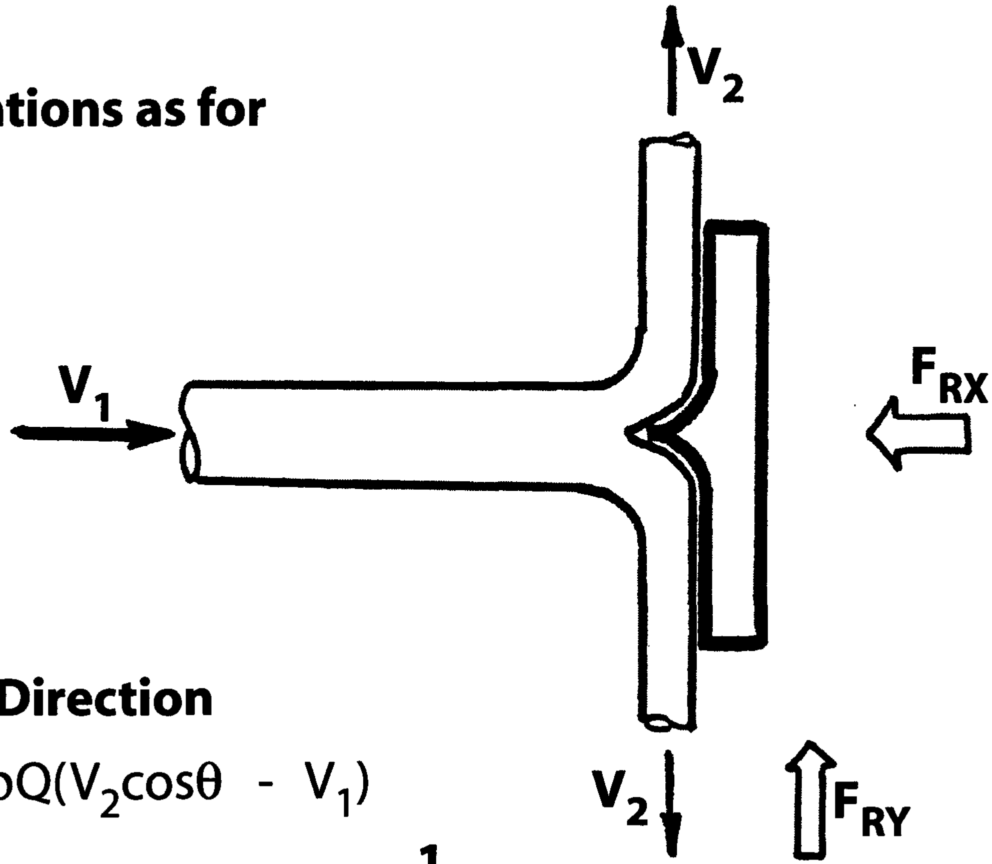
Since p_1 and p_2 are zero

$$F_{RY} = \rho QV_2\sin\theta$$

FORCES IN FREE JETS

FLAT BLADE

Use Same Equations as for Angled Blade



Reaction in X - Direction

$$F_{RX} = -\rho Q(V_2\cos\theta - V_1)$$

For Equally Divided Flow $Q_2 = \frac{1}{2}Q_1$

$$F_{RX} = -\rho\frac{1}{2}Q(-V_1) - \rho\frac{1}{2}Q(-V_1)$$

$$= \rho Q V_1$$

Reaction in Y - Direction

$$F_{RY} = \rho Q V_2\sin\theta$$

For Equally Divided Flow $Q_2 = \frac{1}{2}Q_1$

$$F_{RY} = \rho\frac{1}{2}QV_2 - \rho\frac{1}{2}QV_2$$

$$= 0$$

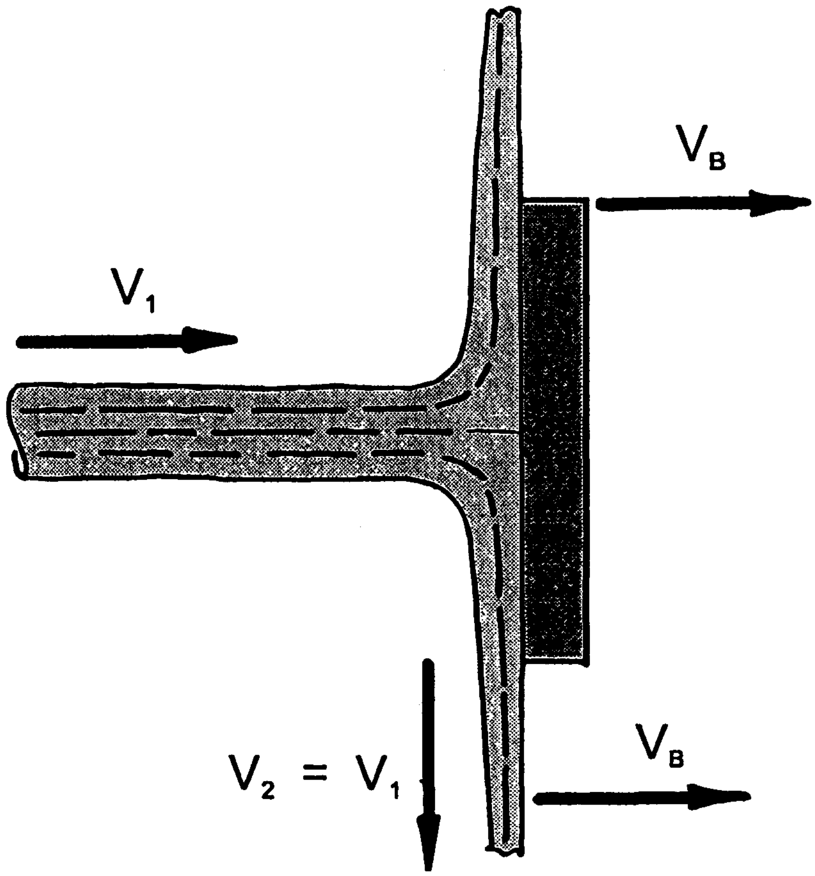
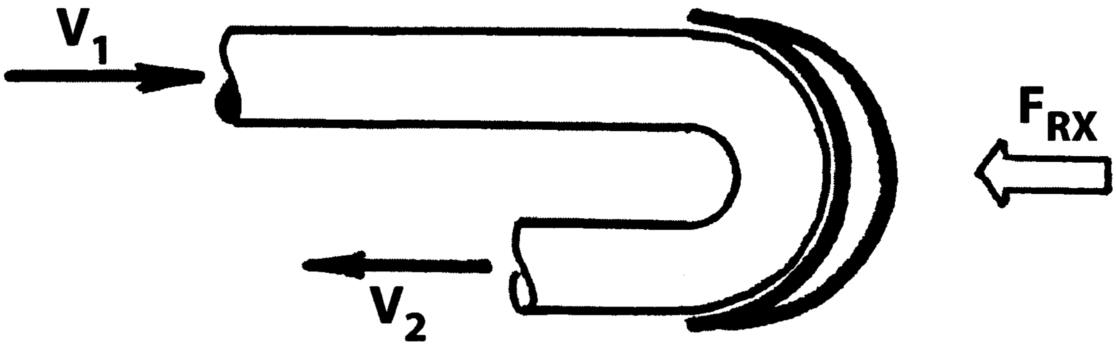


Figure 1 Jet impinging upon a flat moving plate

FORCES IN FREE JETS

U-BLADE



Use Previous Equation

Reaction in X - Direction

$$F_{RX} = -\rho Q(V_2 \cos\theta - V_1)$$

Note that $\cos 180^o = -1$

$$F_{RX} = -\rho Q(-V_2 - V_1)$$

$$= \rho Q(V_2 + V_1)$$

Note that the Energy Equation gives $V_2 = V_1$ for no friction

$$F_{RX} = \rho Q 2V_1$$

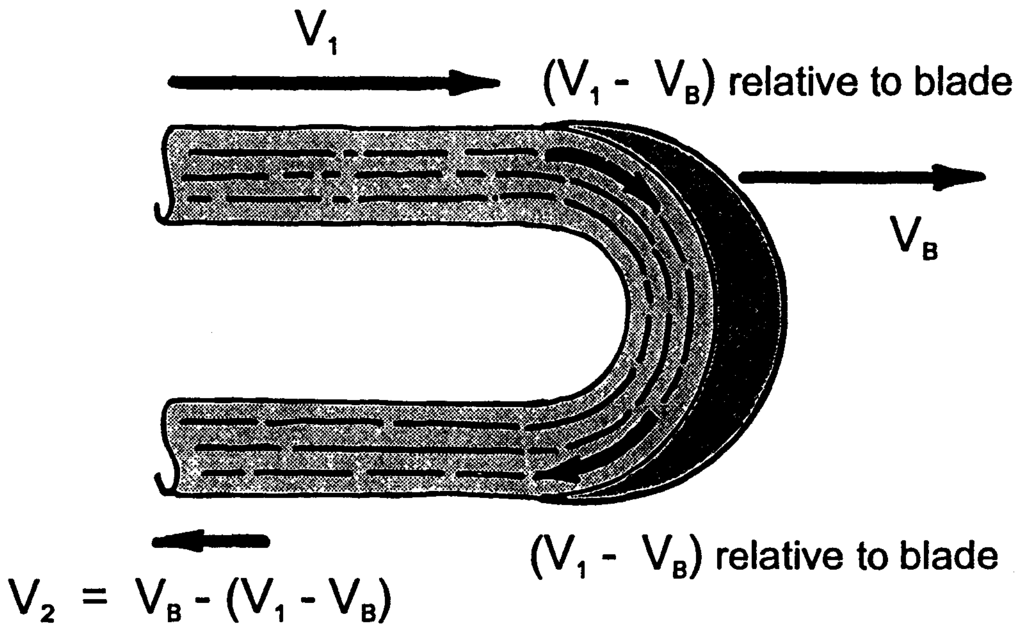
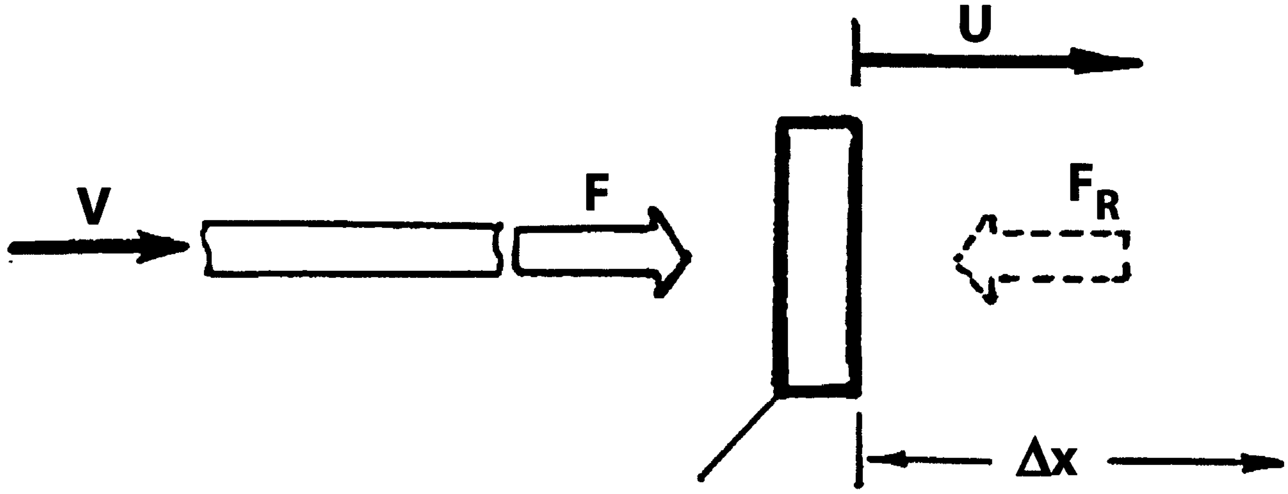


Figure 2 Jet impinging upon a curved moving plate

WORK AND POWER



Force on blade $F = \rho Q \Delta V$ (N)

Work on blade $W = \text{Force} \times \text{Distance}$

$= F\Delta x$

$= \rho Q \Delta V \Delta x$ (Nm)

Power to blade $P = \text{Work} / \text{Time}$

$= F\Delta x / \Delta t$

$= FU$

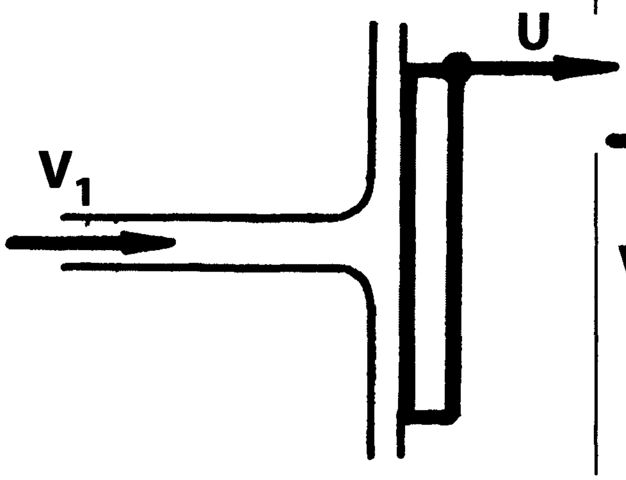
$= \rho Q \Delta V\, U$ (J/s)

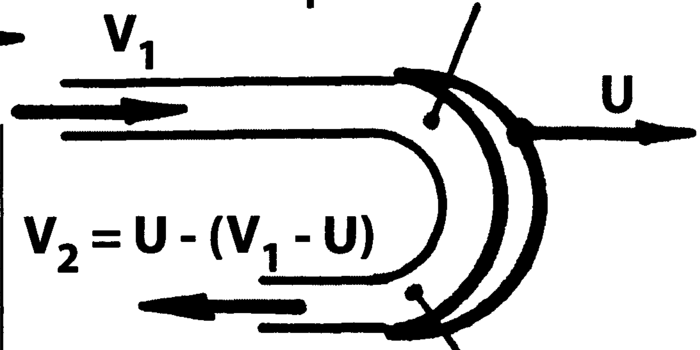
Now since $\Delta V = (V - U)$

Power to blade $P = \rho Q(V - U)U$

IMPULSE POWER

FLAT BLADE | CURVED BLADE





Force on Blade

Flat blade:

$$F = \rho Q(\Delta V)$$
$$= \rho Q(V_1 - U)$$

Curved blade:

$$F = \rho Q(\Delta V)$$
$$= \rho Q[V_1 - \{U - (V_1 - U)\}]$$
$$= \rho Q[V_1 - \{U - V_1 + U\}]$$
$$= \rho Q[2V_1 - 2U]$$
$$= \rho Q2(V_1 - U)$$

Power to Blade

Flat blade:

$$P = \rho Q(V_1 - U)U$$
$$= \rho Q(V_1U - U^2)$$

Curved blade:

$$P = \rho Q2(V_1 - U)U$$
$$= \rho Q2(V_1U - U^2)$$

Blade Speed for Maximum Power $dP/dU = 0$

Flat blade:

$$\frac{dP}{dU} = \rho Q(V_1 - 2U)$$
$$0 = \rho Q(V_1 - 2U)$$
$$V_1 = 2U$$

Curved blade:

$$\frac{dP}{dU} = 2\rho Q(V_1 - 2U)$$
$$0 = 2\rho Q(V_1 - 2U)$$
$$V_1 = 2U$$

For Maximum Power

$$U = \frac{1}{2}V_1$$

$$U = \frac{1}{2}V_1$$

Maximum Power on Blade

$$P = \rho Q(V_1 - \frac{1}{2}V_1)\ V_1$$
$$= \rho Q(\frac{1}{2}V_1)\frac{1}{2}V_1$$
$$= \frac{1}{4}\rho Q V_1^2 \quad \text{(J/s)}$$

$$P = \rho Q 2(\frac{1}{2}V_1)\frac{1}{2}V_1$$
$$= \rho Q 2(\frac{1}{2}V_1)\frac{1}{2}V_1$$
$$= \frac{1}{2}\rho Q V_1^2 \quad \text{(J/s)}$$

Jet Power

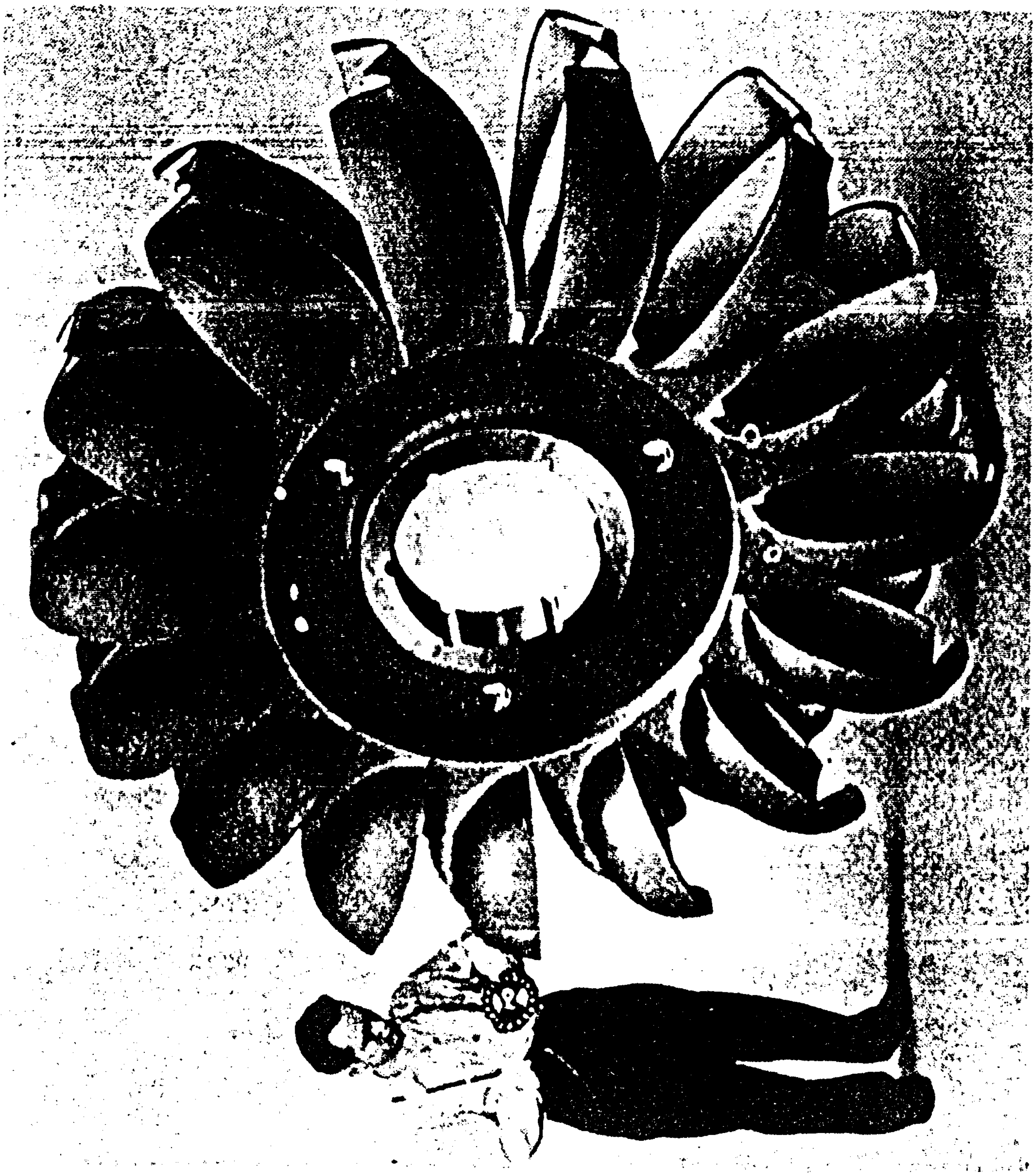
$$E = \frac{1}{2}MV_1^2$$
$$= \frac{1}{2}\rho Q V_1^2 \quad \text{(J/s)}$$

$$E = \frac{1}{2}MV_1^2$$
$$= \frac{1}{2}\rho Q V_1^2 \quad \text{(J/s)}$$

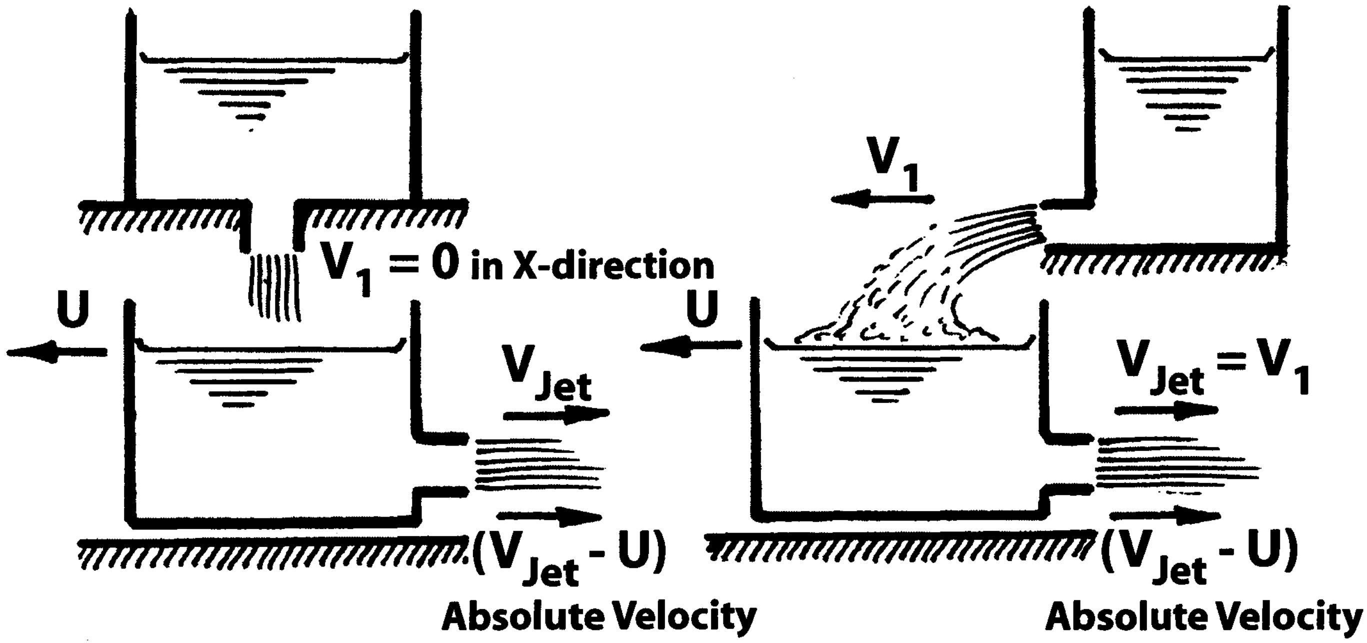
Efficiency = Output / Input

$$\eta = P / E$$
$$= \frac{1}{4}\rho Q V_1^2 \Big/ \frac{1}{2}\rho Q V_1^2$$
$$= \frac{1}{2}$$
$$= 50\%$$

$$\eta = P / E$$
$$= \frac{1}{2}\rho Q V_1^2 \Big/ \frac{1}{2}\rho Q V_1^2$$
$$= 1$$
$$= 100\%$$



REACTION POWER



Force on Bucket

$$F = \rho Q(\Delta V)$$
$$= \rho Q[0 - (V_{Jet} - U)]$$
$$= \rho Q(-V_{Jet} + U)$$

$$F = \rho Q(\Delta V)$$
$$= \rho Q[-V_1 - (V_{Jet} - U)]$$
$$= \rho Q(-V_1 - V_1 + U)$$
$$= \rho Q(-2V_1 + U)$$

Power to Bucket

$$P = F(-U)$$
$$= \rho Q(-V_{Jet} + U)(-U)$$
$$= \rho Q(V_{Jet} - U^2)$$

$$P = F(-U)$$
$$= \rho Q(-2V_1 + U)(-U)$$
$$= \rho Q(2V_1U - U^2)$$

Bucket Speed for Maximum Power dP/dU = 0

$$\frac{dP}{dU} = \rho Q(V_{Jet} - 2U)$$
$$\therefore \ U = \frac{1}{2}V_{Jet}$$

$$\frac{dP}{dU} = \rho Q(2V_1 - 2U)$$
$$\therefore \ U = V_1$$

For Maximum Power

$U = \frac{1}{2}V_{Jet}$

$U = V_1 = V_{Jet}$

Maximum Power on Bucket

$$P = \rho Q(-V_{Jet} + \tfrac{1}{2}V_{Jet})(-\tfrac{1}{2}V_{Jet})$$
$$= \rho Q(-\tfrac{1}{2}V_{Jet})(-\tfrac{1}{2}V_{Jet})$$
$$= \tfrac{1}{4}\rho Q V_{Jet}^2$$

$$P = \rho Q(2V_1 + V_1)(-V_1)$$
$$= \rho Q(-V_1)(-V_1)$$
$$= \rho Q V_1^2$$

Potential Jet Power in Direction of Motion

$$E = \tfrac{1}{2}\rho Q V_{Jet}^2$$

$$E = \tfrac{1}{2}\rho Q V_{Jet}^2 + \tfrac{1}{2}\rho Q V_1^2$$
$$= \tfrac{1}{2}\rho Q V_1^2 + \tfrac{1}{2}\rho Q V_1^2$$
$$= \rho Q V_1^2$$

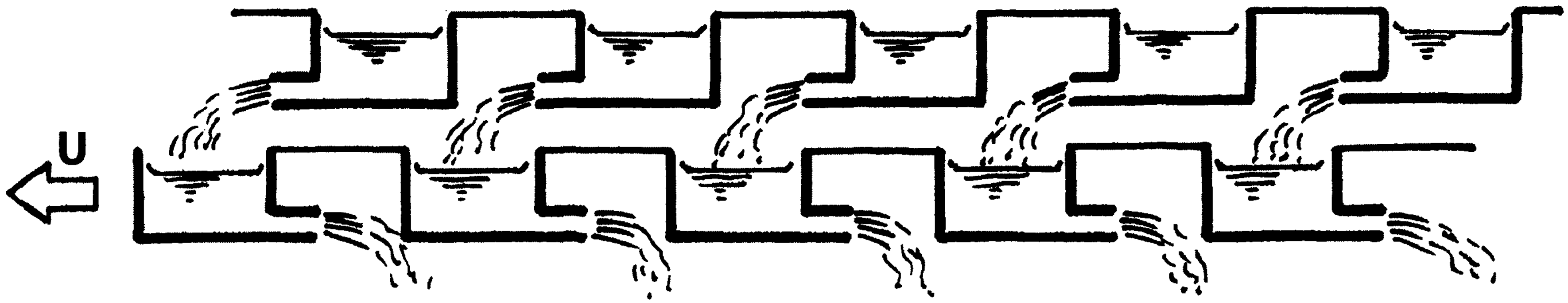
Efficiency = Output / Input

$$\eta = P / E$$
$$= \tfrac{1}{4}\rho Q V_{Jet}^2 \Big/ \tfrac{1}{2}\rho Q V_{Jet}^2$$
$$= \frac{1}{2}$$
$$= 50\%$$

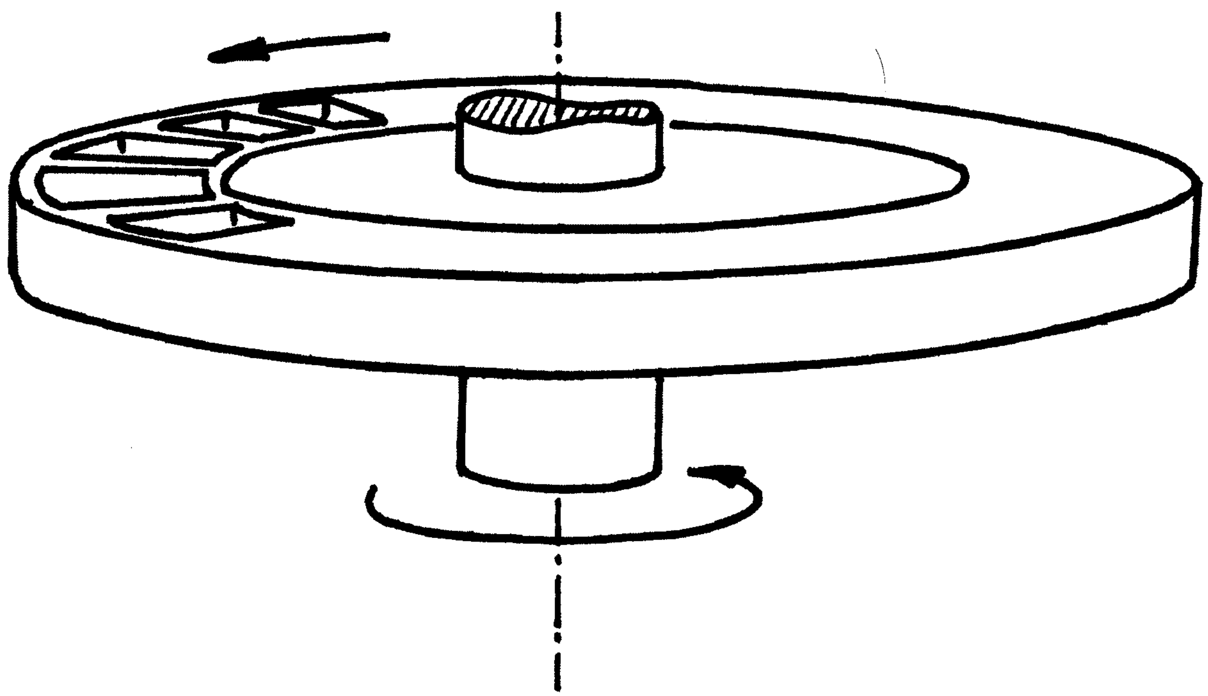
$$\eta = P / E$$
$$= \rho Q V_1^2 / \rho Q V_1^2$$
$$= 1$$
$$= 100\%$$

REACTION TURBINE

Sequence of Fixed and Moving Buckets



Circular Arrangement of Buckets



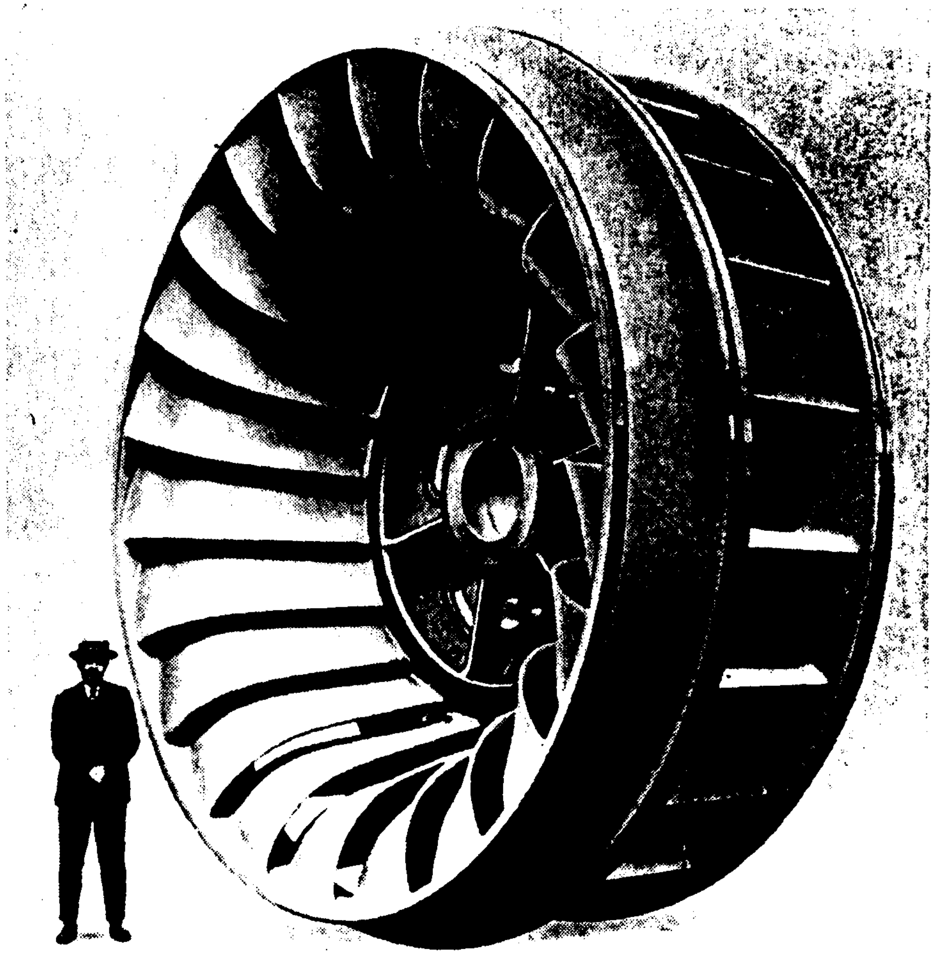


Figure 16.3 Francis runner at Niagara Falls. $h = 214$ ft, $n = 107$ rpm, $\eta = 93.8$ percent at 72,500 hp, diameter = 176 in, overall diameter at band = $183\frac{3}{8}$ in. (*Courtesy of Allis-Chalmers Mfg. Co.*)