SECTION CA

HEAT EXCHANGERS

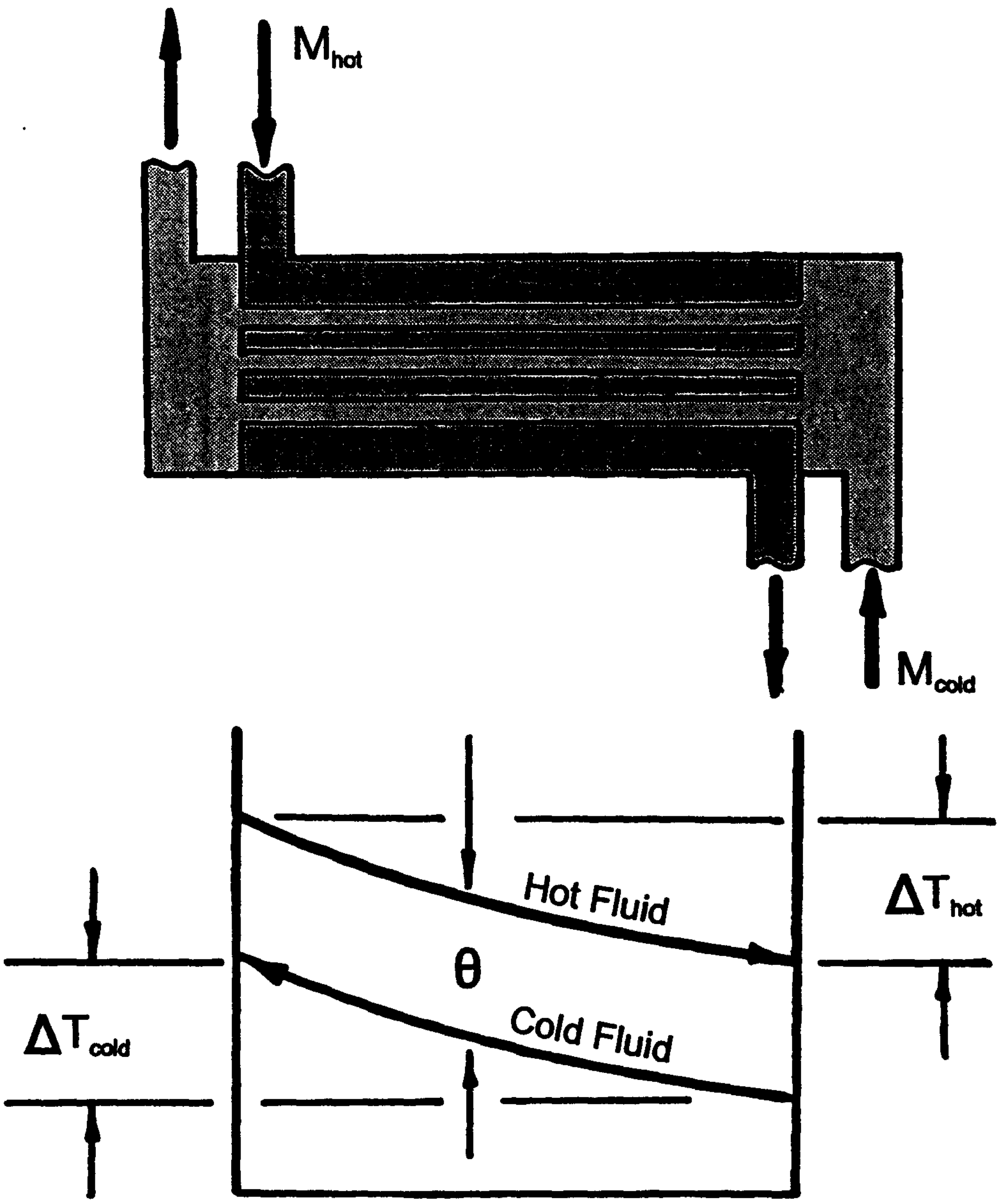


Figure 10 Heat exchanger arrangement and temperature profile

HEAT EXCHANGER TYPES

Heat Exchangers may be divided into three broad categories

- Surface Heat Exchangers
- Contact Heat Exchangers
- Regenerative Heat Exchangers

Regenerative heat exchangers (rotary air heater) operate on the principle of heat storage in a suitable medium and are outside the scope of this course. They are restricted in application and used mainly for gases. Contact heat exchangers operate on the principle of direct contact of the two fluids which mix intimately and share their heat (deaerator, spray condenser, wet cooling tower). For good thermodynamic performance of this type there is inevitably total phase change (deaerator) or partial phase change (wet cooling tower). Also the two fluids must be the same (deaerator) or easily separated (wet cooling tower). Although efficient in operation they are limited in application. Surface heat exchangers are by far the most common due to their versatility in application. Essentially the two fluids are separated by a surface through which the heat is transferred. The surface may be in the form of tubes or plates. Fins may be applied to one surface to enhance the heat transfer by increasing that surface area if this is required by markedly different heat transfer capabilities of the two fluids.

Surface heat exchangers may be classified by the direction of flow of the fluids.

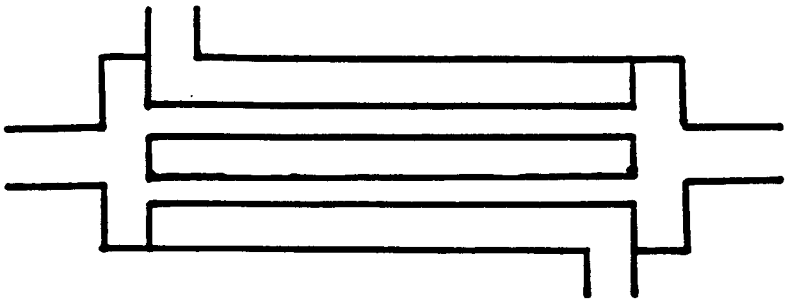
- Parallel Flow
- Counter Flow
- Cross Flow

Since the two fluids are separated by a surface over which they must flow, the flow direction must be considered in any analysis.

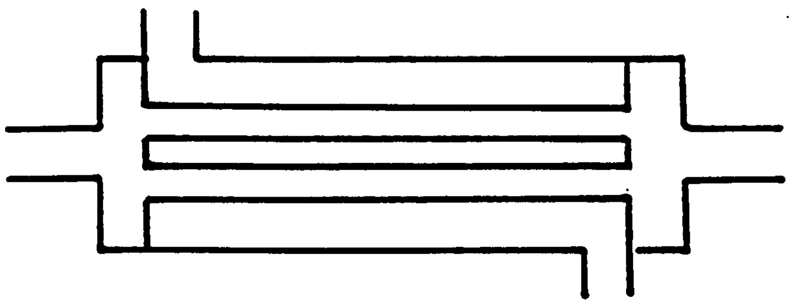
Parallel flow and counter flow configurations are easy to analyse but many heat exchangers are designed for cross flow (car radiator) for practical reasons. Counter flow gives the best thermodynamic performance. Thus for a combination of thermal efficiency and practical considerations the hybrid of cross flow plus counter flow is common. Such combinations are difficult to analyse and empirical relationships based on experimental finding are widely used.

HEAT EXCHANGER CLASSIFICATION

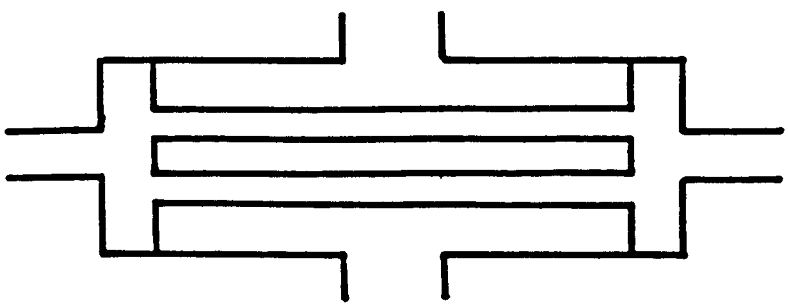
PARALLEL FLOW



COUNTER FLOW



CROSS FLOW



PARALLEL FLOW AND COUNTER FLOW

In parallel flow heat exchangers the two fluids flow parallel to one another in the same direction. The temperature difference Θ between the two fluids becomes smaller as the two fluids exchange heat and the final temperatures tend to converge on some value between the two initial values. This is well illustrated on a T-L Diagram.

In counter flow heat exchangers the two fluids flow parallel to one another but in opposite directions. The temperature difference Θ between the two fluids tends to be the same but may converge or diverge slightly. A characteristic of this configuration is that the final temperature of the cold fluid may be higher than the final temperature of the hot fluid. This is also well illustrated on a T-L Diagram provided the direction of flow is indicated.

An analysis of the temperature difference prevailing between the two fluids will be based on these two configurations. Heat transferred Q from the hot fluid to the cold fluid is given by the following equation

$$Q = UA\Theta$$

where Θ is the temperature difference and U is the overall heat transfer coefficient.

It is immediately evident that for a given value of heat transfer Q a small value of surface area A will give low capital cost but a small value of Θ will give good thermodynamic performance (small Θ gives small increase in entropy, small increase in unavailable energy and large effectiveness of transfer of available energy). If the overall heat transfer coefficient U is dictated by fluid properties and flow conditions the design of the heater becomes a compromise between capital cost (dictated by surface area A) and thermal efficiency (dictated by temperature difference Θ)

TEMPERATURE DIFFERENCE

If the fluids in passing through the heat exchanger give up their heat in such a way that the plot of temperature on a T-A Diagram is a straight line, then the effective temperature difference between the two fluids is simply the average temperature difference.

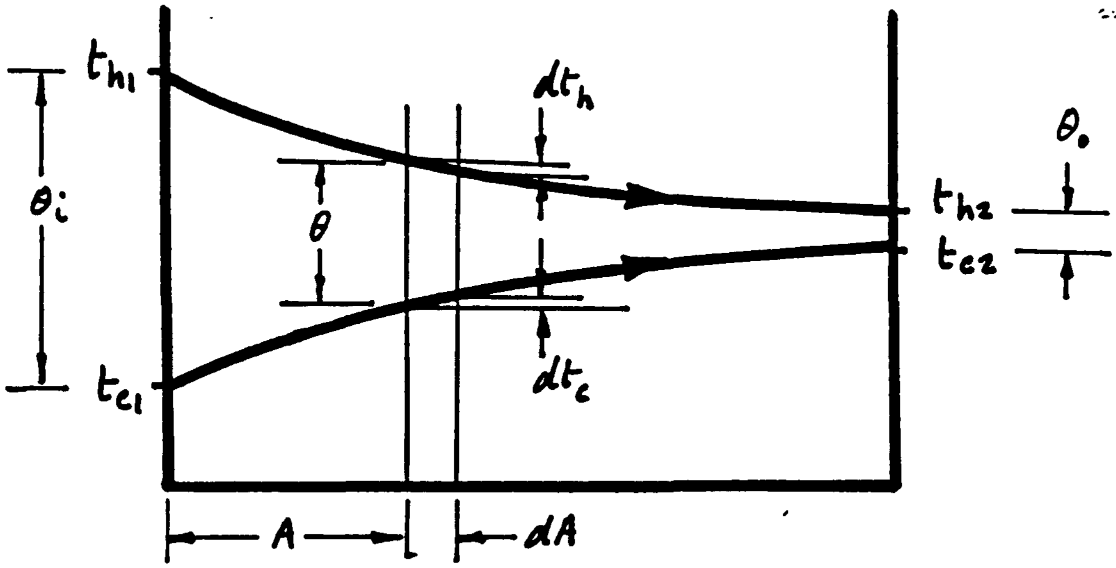
$$\Theta_a = (\Theta_o + \Theta_i) / 2$$

If however the fluids give up more heat in certain parts of the heat exchanger the plot will not be linear and a more detailed analysis is required.

The log mean temperature difference takes account of the fact that the that exchanged between the two fluids depends upon the temperature difference prevailing at each point within the heat exchanger. The log mean temperature difference is derived to give the following:

$$\Theta_m = (\Theta_o - \Theta_i)/Ln\ (\Theta_o/\Theta_i)$$

LOG MEAN TEMPERATURE
PARALLEL FLOW



$$dQ = U_A \, dA \, \theta \qquad \text{------------------} \quad (1)$$

$$dQ = m_c \, c_{pc} \, dt_c$$

$$dQ = m_h \, c_{ph} \, dt_h$$

NOTE THAT dt_h DECREASES WITH INCREASING A AND IS MATHEMATICALLY NEGATIVE

$$dt_c = \frac{dQ}{m_c c_{pc}} \qquad \text{------------------} \quad (2)$$

$$dh_h = - \frac{dQ}{m_h c_{ph}} \qquad \text{------------------} \quad (3)$$

$$\theta = t_h - t_c$$

$$d\theta = dt_h - dt_c \qquad \text{------------------} \quad (4)$$

SUBSTITUTE (2) AND (3) INTO (4)

$$d\theta = -\frac{dQ}{m_h c_{ph}} - \frac{dQ}{m_c c_{pc}}$$

$$d\theta = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) dQ \quad \text{------------ (5)}$$

$$\int_i^o d\theta = -\int_i^o \left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) dQ$$

$$\theta_o - \theta_i = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) Q$$

$$\frac{\theta_o - \theta_i}{Q} = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) \quad \text{------------ (6)}$$

SUBSTITUTE (1) INTO (5)

$$d\theta = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) U_A \, dA \, \theta$$

$$\frac{d\theta}{\theta} = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) U_A \, dA$$

$$\int_i^o d\theta = -\int_i^o \left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) U_A \, A$$

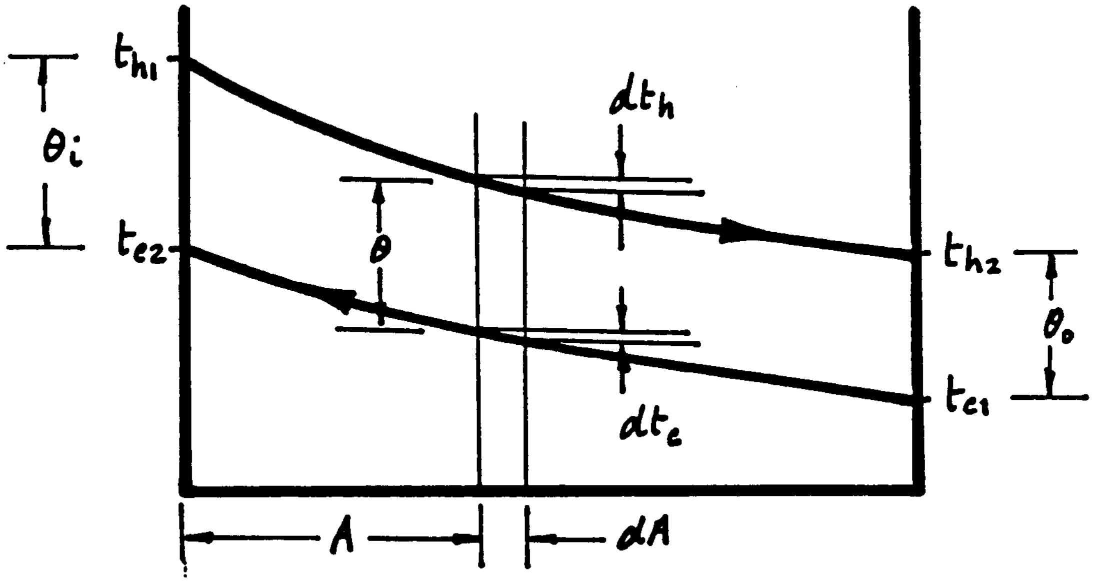
$$\mathrm{Ln}\left(\frac{\theta_o}{\theta_i}\right) = -\left(\frac{1}{m_h c_{ph}} + \frac{1}{m_c c_{pc}}\right) U_A \, A \quad \text{-------- (7)}$$

SUBSTITUTE (6) INTO (7)

$$\mathrm{Ln}\left(\frac{\theta_o}{\theta_i}\right) = \frac{\theta_o - \theta_i}{Q} U_A \, A$$

$$\boxed{Q = U_A \, A \, \frac{\theta_o - \theta_1}{\mathrm{Ln}(\theta_o / \theta_1)}}$$

LOG MEAN TEMPERATURE
COUNTER FLOW



$$dQ = U_A \, dA \, \theta \qquad (1)$$

$$dQ = m_c \, c_{pc} \, dt_c$$

NOTE THAT dt_c DECREASES WITH INCREASING A AND IS MATHEMATICALLY NEGATIVE

$$dQ = m_h \, c_{ph} \, dt_h$$

NOTE THAT dt_h DECREASES WITH INCREASING A AND IS MATHEMATICALLY NEGATIVE

$$dt_c = -\frac{dQ}{m_c c_{pc}} \qquad (2)$$

$$dt_h = -\frac{dQ}{m_h c_{ph}} \qquad (3)$$

$$\theta = t_h - t_c$$

$$d\theta = dt_h - dt_c \qquad (4)$$

SUBSTITUTE (2) AND (3) INTO (4)

$$d\theta = -\frac{dQ}{m_h c_{ph}} + \frac{dQ}{m_c c_{pc}}$$

$$d\theta = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) dQ \quad \text{------------ (5)}$$

$$\int_i^o d\theta = -\int_i^o \left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) dQ$$

$$\theta_o - \theta_i = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) Q$$

$$\frac{\theta_o - \theta_i}{Q} = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) \quad \text{------------ (6)}$$

SUBSTITUTE (1) INTO (5)

$$d\theta = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) U_A \, dA \, \theta$$

$$\frac{d\theta}{\theta} = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) U_A \, dA$$

$$\int_i^o d\theta = -\int_i^o \left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) U_A \, A$$

$$\mathrm{Ln}\left(\frac{\theta_o}{\theta_i}\right) = -\left(\frac{1}{m_h c_{ph}} - \frac{1}{m_c c_{pc}}\right) U_A \, A \quad \text{-------- (7)}$$

SUBSTITUTE (6) INTO (7)

$$\mathrm{Ln}\left(\frac{\theta_o}{\theta_i}\right) = \frac{\theta_o - \theta_i}{Q} U_A \, A$$

$$\boxed{Q = U_A \, A \, \frac{\theta_o - \theta_1}{\mathrm{Ln}(\theta_o / \theta_1)}}$$

HEAT EXCHANGER EFFECTIVENESS

A heat exchanger of increasing size or surface are A will result in a reducing Θ and an increasing amount of heat transferred Q assuming that the overall heat transfer coefficient U remains constant since

$$Q = UA\Theta$$

The question which now arises is what are the relative effects on Q and Θ of a change in A? Which is more significant and why? This depends upon the heat capacity of each fluid. Assuming a constant flow of each fluid the limit of Q is reached when one or other of the fluids has changed its temperature by the maximum amount ΔT since

$$Q = M_{cp}\Delta T$$

This sets a limit on Q and any other increase in A will only affect Θ.

The effectiveness of a heat exchanger is based on this maximum value of Q which in turn is defined by ΔT for one or other of the fluids. The effectiveness is thus defined as the ratio of the energy actually transferred to the maximum theoretically possible.

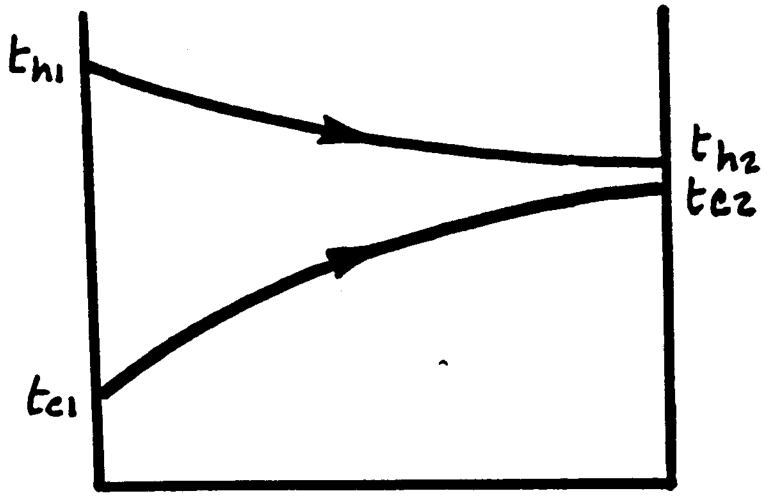
Effectiveness = Actual Energy Transfer / Theoretical Maximum Transfer

Analysed in terms of temperatures the effectiveness E is given by

$$E = (t_{c2} - t_{c1}) / (t_{h1} - t_{c1}) \text{ when } m_h c_{ph} > m_c c_{pc}$$

$$E = (t_{h1} - t_{h2}) / (t_{h1} - t_{c1}) \text{ when } m_h c_{ph} < m_c c_{pc}$$

EFFECTIVENESS
PARALLEL FLOW



$$Q = m_h c_{ph} (t_{h1} - t_{h2})$$

$$Q = m_c c_{pc} (t_{c2} - t_{c1})$$

EFFECTIVENESS IS ACTUAL HEAT TRANSFER / MAXIMUM POSSIBLE HEAT TRANSFER

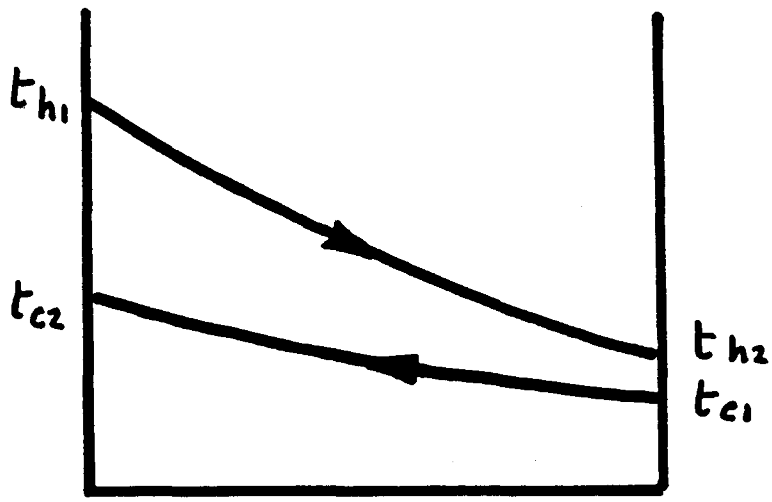
$$E_h = \frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}} \quad \text{FOR} \quad m_c c_{pc} > m_h c_{ph}$$
$$(t_{h1} - t_{h2}) > (t_{c2} - t_{c1})$$

$$E_c = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} \quad \text{FOR} \quad m_h c_{ph} > m_c c_{pc}$$
$$(t_{c2} - t_{c1}) > (t_{h1} - t_{h2})$$

GENERALLY

$$E = \frac{\Delta t \text{ OF FLUID WITH LARGEST TEMPERATURE CHANGE}}{\text{DIFFERENCE OF INLET TEMPERATURES}}$$

EFFECTIVENESS
COUNTER FLOW



$$Q = m_h c_{ph} (t_{h1} - t_{h2})$$

$$Q = m_c c_{pc} (t_{c2} - t_{c1})$$

EFFECTIVENESS IS ACTUAL HEAT TRANSFER / MAXIMUM POSSIBLE HEAT TRANSFER

$$E_h = \frac{t_{h1} - t_{h2}}{t_{h1} - t_{c1}} \quad \text{FOR } m_c c_{pc} > m_h c_{ph}$$

$$E_c = \frac{t_{c2} - t_{c1}}{t_{h1} - t_{c1}} \quad \text{FOR } m_h c_{ph} > m_c c_{pc}$$

GENERALLY

$$E = \frac{\Delta t \text{ OF FLUID WITH LARGEST TEMPERATURE CHANGE}}{\text{DIFFERENCE OF INLET TEMPERATURES}}$$

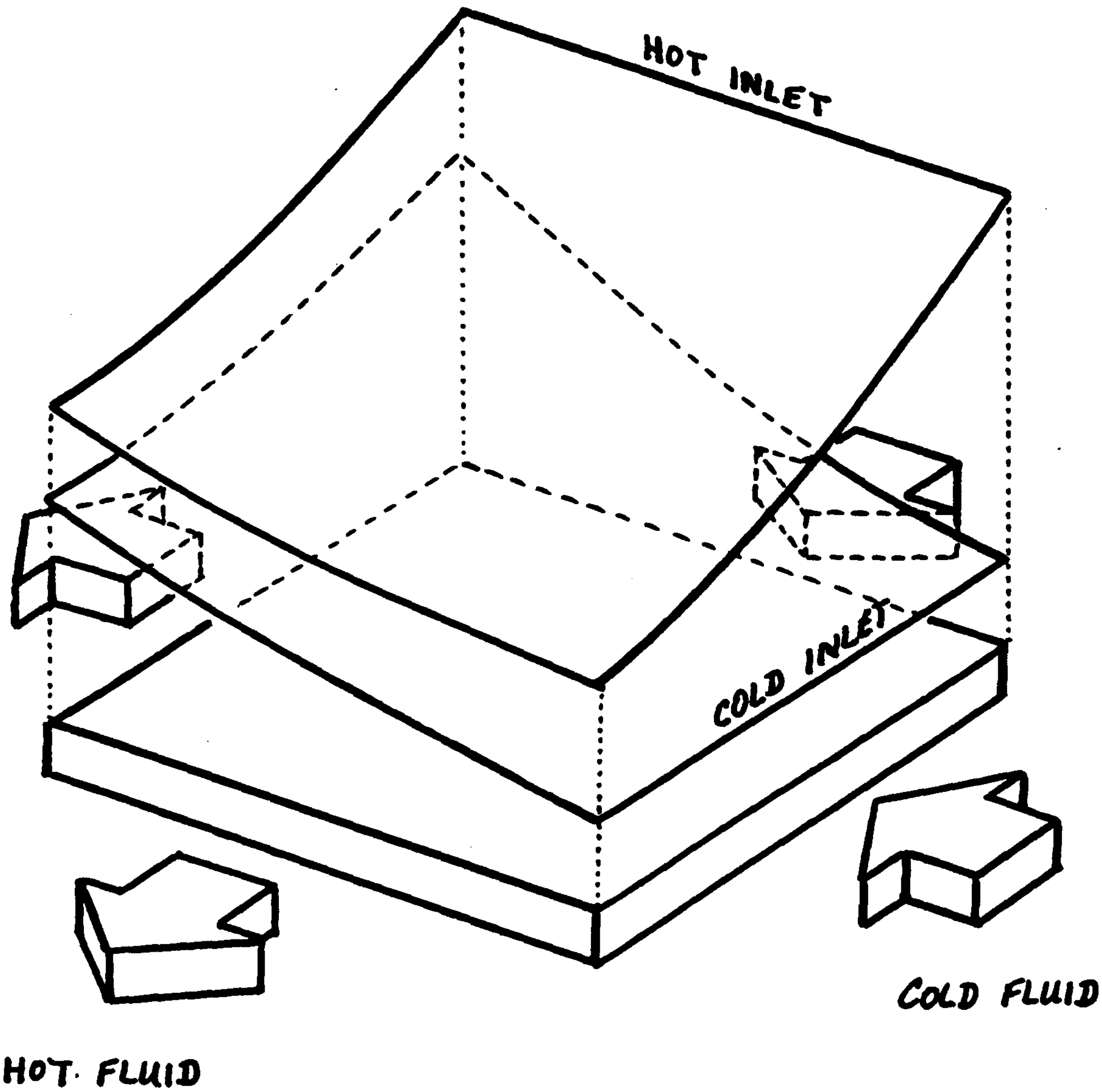
CROSS FLOW

Cross flow or transverse flow heat exchangers are common in many practical applications. One advantage is that the fluid path inlets and outlets are well separated leading to a convenient design. Another advantage is that improved mixing of one or sometimes both fluids can be achieved thus improving the heat flow between that fluid and the heat exchange surface. This is particularly true of a fluid flowing across the outside of a tube bank. Mathematical analysis of such flows is very difficult and much design is based on empirical relationships. Thus correction factors are available and can be applied as follows:

$$Q = UAF\Theta_m$$

where F is a correction factor for a particular configuration and conditions. Under certain conditions it is possible for F to be slightly greater than unity if the configuration actually creates an improvement in heat transfer over that of a single tube. Normally F is less than unity since the heat transfer of a tube in a tube bundle is less than that of single tube.

CROSS FLOW



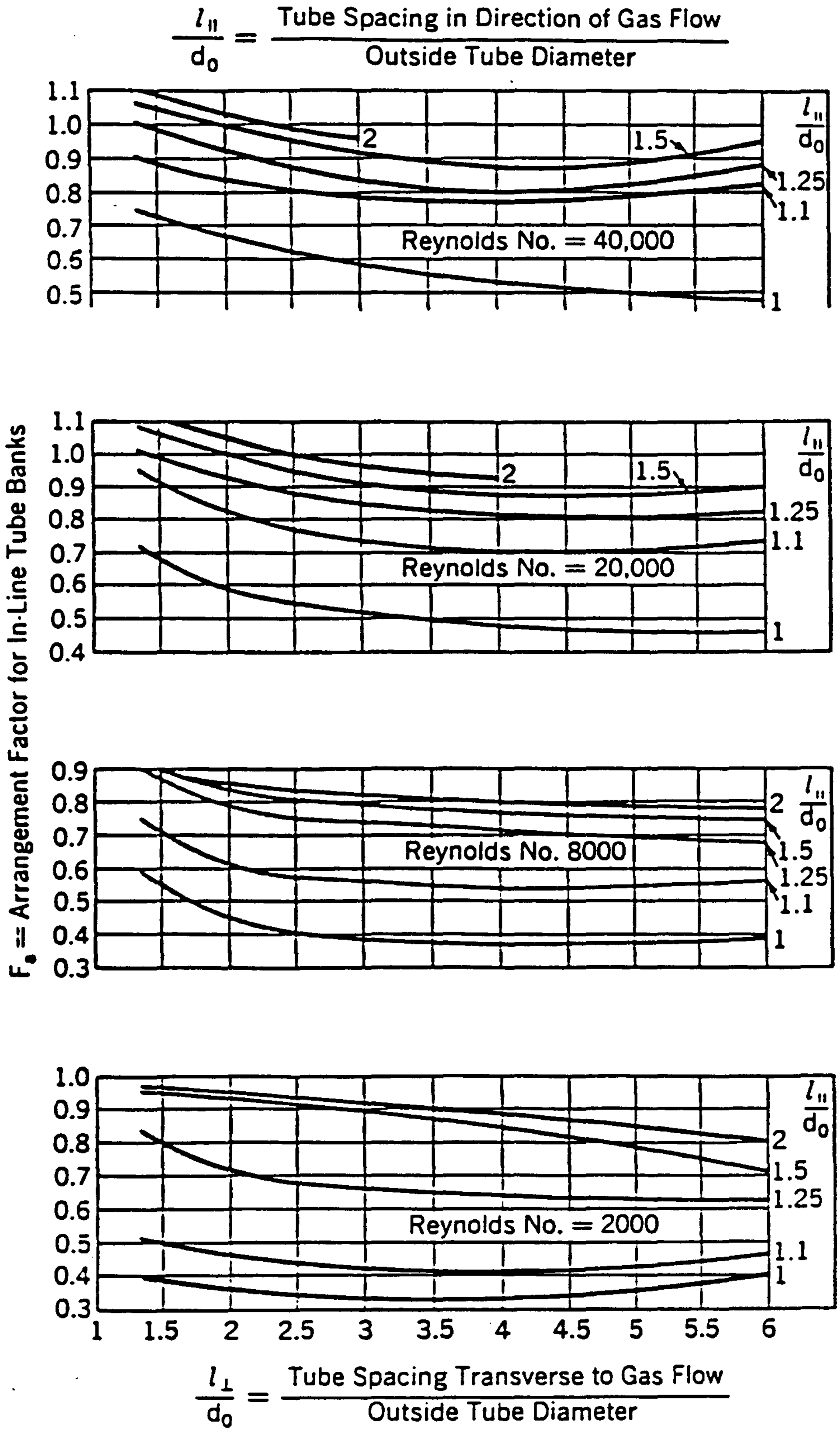


Fig. 14 Arrangement factor, F_a, as affected by Reynolds number for various in-line tube patterns, crossflow gas or air.

HEAT TRANSFER CALCULATIONS

The general equation for heat transfer in a heat exchanger is

$$Q = UA\Delta T$$

The area A is simply the heat transfer surface area of the heat exchanger. ΔT or θ is the temperature difference between the two fluids and is usually calculated as the log mean temperature difference. This leaves the overall heat transfer coefficient U to be determined.

The overall heat transfer coefficient is made up of three components:

- h_{FLUID1} heat transfer coefficient on one side of wall.
- k_{WALL} thermal conductivity of wall.
- h_{FLUID2} heat transfer coefficient on other side of wall.

These are combined as follows to obtain the overall heat transfer coefficient:

$$\frac{1}{U} = \frac{1}{h_{FLUID\,1}} + \frac{\ell}{k_{WALL}} + \frac{1}{h_{FLUID2}}$$

where ℓ is the thickness of the wall.

For the thermal conductivity term k is a property of the material and can be obtained from suitable references. A complication arises if the wall is not plane (flat). For circular tubes a correction incorporating the inner and outer radii must be applied.

The value of the heat transfer coefficient h depends upon both the fluid properties and the flow conditions. Flow is usually turbulent having irregular oscillations. A pure mathematical analysis is thus almost impossible and empirical relations obtained by experiment must be used.

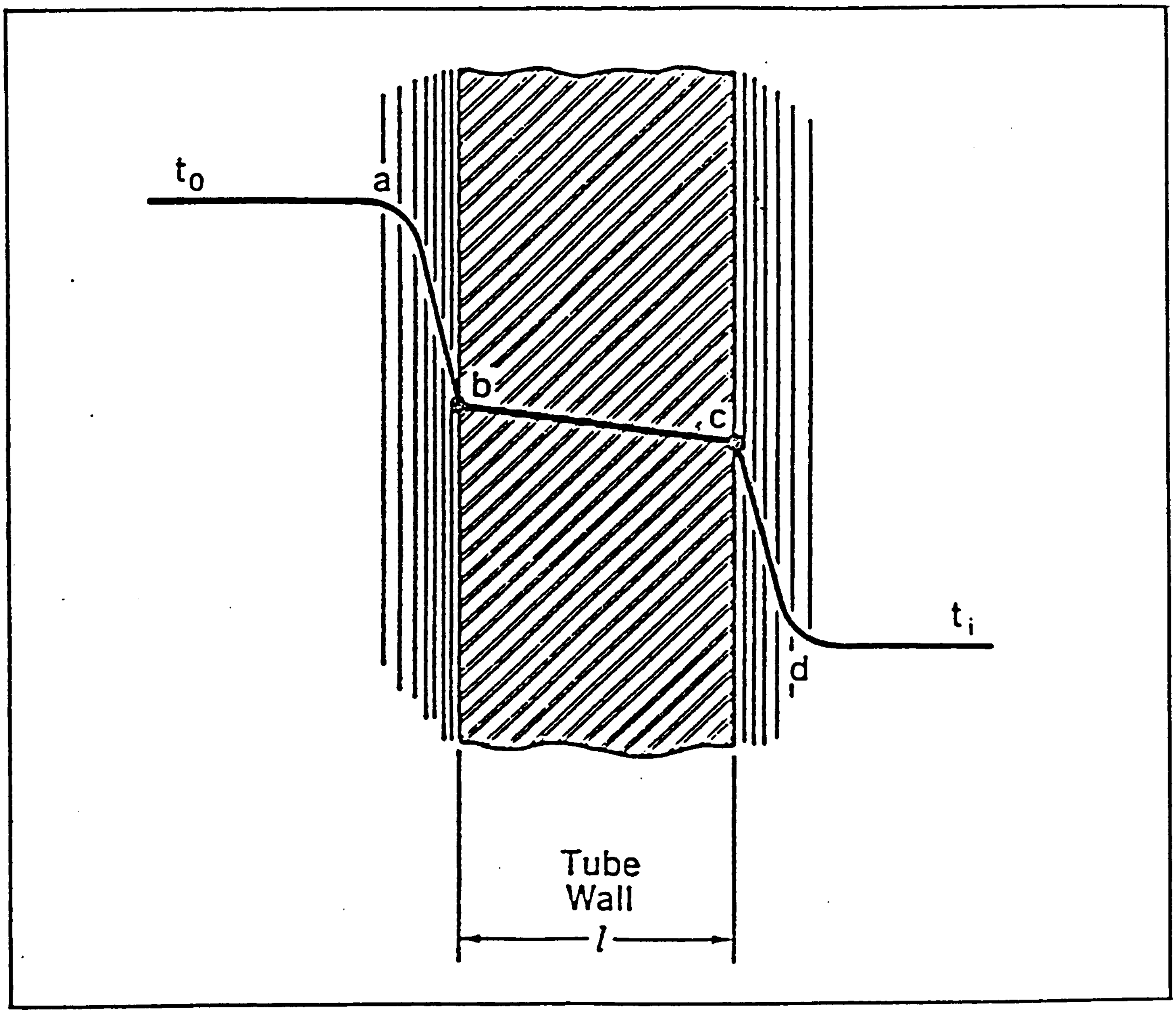


Fig. 16 Temperature gradients through fluid films and tube wall separating two fluids.

HEAT TRANSFER COEFFICIENT

The heat transfer coefficients on the inside and outside of the tubes is determined by the use of certain non-dimensional numbers:

Re Reynolds Number
Pr Prandtl Number
Gr Grashof Number
Nu Nusselt Number

These are defined in terms of the fluid properties, flow conditions or physical parameters:

$$Re = \frac{\rho V D}{\mu}$$ (inertia and viscous effects)

$$Pr = \frac{c_p \mu}{k}$$ (fluid thermal properties)

$$Gr = \frac{\beta g \theta \rho^2 l^3}{\mu^2}$$ (buoyancy and viscous effects)

$$Nu = \frac{h d}{k}$$ (heat transfer properties)

They are also related to one another by empirical equations some of which are given below:

Convective Heat Transfer inside a Pipe

$N_u = 0.023\ (R_e)^{0.8}\ (P_r)^{0.4}$ for heating ($T_{PIPE} > T_{FLUID}$)

$N_u = 0.023\ (R_e)^{0.8}\ (P_r)^{0.33}$

$N_u = 0.023\ (R_e)^{0.8}\ (P_r)^{0.3}$ for cooling ($T_{PIPE} < T_{FLUID}$)

Convective Heat Transfer across a Tube Bank

$N_u = F_a\ 0.287\ (Re)^{0.6}\ (Pr)^{0.33}$

F_a = ARRANGEMENT FACTOR

Condensing Heat Transfer in a Tube Bank

$$N_u = \left(\frac{h_{fg} g \rho^2 d^3}{4 \mu k \theta}\right)^{0.25}$$

When executing a calculation the first step is to determine all the fluid properties at the appropriate temperature. The next step is to calculate the Nusselt Number Nu since this incorporates the heat transfer coefficient h. The final step is to evaluate h.

Note that, since the relationship between the non-dimensional numbers is empirical, different textbooks give slightly different equations.