Problem Set 2

UN0701 – Engineering Risk and Reliability — Summer 2005

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1. Consider a simple *demand* vs. *capacity* relationship, $Z = R - S$, where R is the capacity (or resistance) and S is the demand (or loading). Assume that capacity is described by the NORMAL distribution with $\mu_R = 200$, $\sigma_R = 40$, and the demand is also described by the NORMAL distribution with $\mu_S = 100$, $\sigma_S = 30$.

   a) What is the distribution of Z ?
   b) Estimate the mean and standard deviation of Z.
   c) Calculate the reliability index β and the probability of failure (i.e. $P[Z \leq 0]$).
   d) Compute the probability of failure using Monte Carlo simulation and compare the results to part (c). Approximately how many MCS trials are required to get a reasonable estimate of the probability of failure? Comment on the results.

2. Consider again the *demand* vs. *capacity* relationship, $Z = R - S$, where R is $N \equiv (\mu_R, \sigma_R)$ and the capacity S is $N \equiv (\mu_S, \sigma_S)$. Assuming a probability of failure for design equal to 10^{-4} and a resistance factor ϕ of 0.9, plot the load factor α against the coefficient of variation for the resistance. Assume that the mean capacity is 100 with a COV of 0.3.

3. Consider the following limit-state equation in STANDARD NORMAL space.

   $$G(y_1, y_2) = y_1 + 3y_2 - 6 = 0$$

   Calculate the reliability index β. Plot the design point and the limit-state equation.

4. Consider the following limit-state equation in STANDARD NORMAL space.

   $$G(y_1, y_2) = (y_1 - 2)^2 - 3y_2 + 6 = 0$$

   Calculate the reliability index β using the SOLVER function in Excel. Plot the design point and the limit-state equation.

5. Suppose that the PDF of a random variable X is as follows:

   $$f_X(x) = \begin{cases} e^{-x} & \text{for } x > 0 \\ 0 & \text{otherwise} \end{cases}$$

   Given that another random variable Y is described as $Y = \sqrt{X}$

   a) Determine the PDF of Y.
   b) Derive a second-order approximation to the mean of Y.

6. A variable Y is a quadratic polynomial function of a WEIBULL distributed random variable, X, given as

   $Y = a_0 + a_1 X + a_2 X^2$

   The distribution of X is $F(x) = 1 - \exp(-x^{\beta})$, where $\beta = 2$ and the scale parameter is one. Note that the n^{th} moment of the WEIBULL variable is

   $$\mu_n = \int_0^{\infty} x^n f(x)dx = \Gamma\left(1 + \frac{n}{\beta}\right) \qquad n = 1, 2, \cdots$$

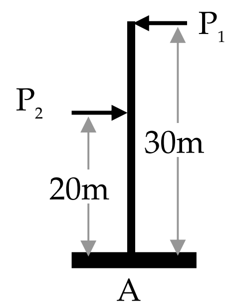
   Find the *mean* and the *standard deviation* of Y in terms of the moments of X using the following two methods:

   a) Taylor series expansion of Y about its mean value (μ_y)
   b) Formal definition of the expectations $E[Y]$ and $E[(Y - \mu_y)^2]$

7. A pole is subjected to two loads P_1 and P_2 so that the bending moment at the bottom of the pole is

   $M_A = 30P_1 - 20P_2$ (kN·m)

   Here, P_1 is a NORMAL random variable with mean 50 kN and standard deviation 5 kN; P_2 is another NORMAL random variable with mean 20 kN and standard deviation 3 kN.



   a) Determine the mean and standard deviation of the bending moment at the bottom of the pole.
   b) If the moment-resisting capacity at the bottom of the pole is M_R, a NORMAL random variable with a mean of 1750 kN·m and a standard deviation of 150 kN·m what is the probability that the pole will fail under the loads P_1 and P_2? What is the associated reliability index?

   Assume all three random variables are independent.

8. The failure strength of a chain depends on its weakest link. It turns out that the distribution of the strength of the weakest link in a long chain follows a WEIBULL distribution. If a particular chain manufacturer knows that $\gamma = 0.5$ and that 90% of their chains have breaking strengths greater than $12{,}000$ lbs, what then is their *median* chain strength?

9. In a testing program, 10 components were tested at $170°$C. The components were periodically inspected for failure. The time to failure data (in hours) are given below:

   $1764,\ 2772,\ 3444,\ 3542,\ 3780,\ 4860,\ 5196,\ 5448+,\ 5448+,\ 5448+$

   Estimate the parameters assuming that the time to failure follows the exponential distribution (one parameter).

10. The time to failure of a component follows the WEIBULL distribution with shape parameter equal to 0.5 and scale parameter equal to 10 years. What is the probability that the component
    a) will survive 5 years without failure?
    b) will fail in the time interval 5-10 years?

11. The constant failure rate of a component is 1 failure per 1000 hours of operation. The time to failure distribution follows the exponential distribution.
    a) What is the reliability for a mission time of 200 hours, given that the current age of component is 100 hours?
    b) What is the mission reliability for a 200 hour operation, if the component age is 250 hours?