

## Problem Set 1

UN0701 – Engineering Risk and Reliability

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**Due:** July 8, 2005

1. When we roll a pair of balanced dice, what are the probabilities of getting
  - a) 7
  - b) 11
  - c) 7 or 11
  - d) 3
  - e) 2 or 12
  - f) 2, 3, or 12?
2. Which of the following is more probable:
  - a) getting at least one six with 4 throws of a die, or
  - b) getting at least one double six with 24 throws of two dice?
3. Several cracks have been discovered in a number of outlet feeder pipes at a CANDU nuclear power plant. Suppose that in a group of 95 outlet feeders, 8 have actually developed through-wall cracks.
  - a) How many ways are there to select a sample of 5 feeders from the group of 95 for a thorough inspection?
  - b) In how many ways can a random sample of 5 feeders contain exactly 4 with cracks? What is the probability that exactly 4 will have cracks?
4. The probability that a wind strong enough to structurally damage the CN tower in Toronto occurring at least once in the next 50 years is estimated to be 0.2. Similarly, the probability that a strong enough earthquake to cause damage to the tower occurring at least once in the next 50 years is estimated to be 0.1. If the tower is damaged by an earthquake in the next 50 years, the probability that it will also be damaged by wind is estimated to be 0.7.
  - a) What is the probability that the tower will be damaged in the next 50 years?
  - b) If the tower is damaged at the end of 50 years, what is the probability that it was damaged both by wind and earthquake?
5. An underground site ( $X$ ) in the Canadian Shield is being considered for the storage of high-level nuclear waste. Within the next 100 years, there is a 1% chance that the hazardous material could leak outside of the storage containment. Two adjacent towns,  $A$  and  $B$ , rely on groundwater for their water supply. The water to each town would be contaminated if there is a leakage in the waste storage and if there exists a continuous pathway (i.e. active fracture network) between the storage containment and the given town. Observe that the presence of a continuous pathway would allow the contaminants to move freely and quickly access the region.

Suppose there is a 2% chance of a continuous pathway from the site to  $A$ , and the probability of a continuous pathway to  $B$  is slightly higher and equals 3%. However, if indeed a continuous pathway exists between  $X$  and  $A$ , the probability of continuous pathway between  $X$  and  $B$  is increased to 20%. You may assume the event of leakage from the storage is independent of the presence of fractures. Consider the period over the next 100 years.

- a) What is the probability that water in town  $A$  will be contaminated?
- b) What is the probability that water in at least one of the two towns will be contaminated?

6. If a random variable has the probability density

$$f(x) = \begin{cases} 2e^{-2x} & \text{for } x > 0 \\ 0 & \text{for } x \leq 0 \end{cases}$$

find the probabilities that it will take on a value

- a) between 1 and 3
  - b) greater than 0.5
7. A random variable has a NORMAL distribution with the location parameter equal to 62.4. Find its standard deviation if the probability is 0.20 that it will take on a value greater than 79.2.
8. The time between severe earthquakes at a given region follows a LOG-NORMAL distribution with a coefficient of variation of 40%. The expected time between severe earthquakes is 80 years.
- a) Determine the parameters of this LOG-NORMALLY distributed interval time  $T$ .
  - b) Determine the probability that a severe earthquake will occur within 20 years from the previous one.
  - c) Suppose the last severe earthquake in the region took place 100 years ago. What is the probability that a severe earthquake will occur over the next year?
9. Consider a simple *income vs. expenses* relationship,  $R = I - E$ , where  $R$  is the total profit for the year,  $I$  is the income, and  $E$  represents the expenses. Assuming that income can be described using the WEIBULL distribution with  $\mu_I = 60,000$ ,  $\alpha_I = 6,000$ ,  $\gamma_I = 2$ , and the expenses can be modelled using the NORMAL distribution with  $\mu_E = 50,000$ , and  $\sigma_E = 10,000$ ,
- a) Plot the density functions for the income and expenses.
  - b) Construct the histogram and compute the sample statistics for the total profit  $R$  using Monte Carlo simulation. Perform the analysis using 100, 1,000 and 10,000 trials.
  - c) Using the results from the Monte Carlo simulation, estimate the probability of making a profit (i.e.  $P [ R \geq 0 ]$ )

10. Consider the following 4 datasets:

#### Wind Data

The yearly maximum wind speed, in miles per hour, registered at a given location during 50 years is given below. We assume here that this data will be used to determine a design wind speed for structural building purposes. Important facts to be taken into account for these data are its non-negative character and, perhaps, the existence of a non-clearly defined upper bound (the maximum conceivable wind speed is bounded).

22.64	22.80	23.75	24.01	24.04	24.24	24.74	25.45	25.55	25.66
25.99	26.63	26.69	26.88	26.89	27.12	27.43	27.69	27.71	28.12
28.58	28.88	29.12	29.45	29.48	30.18	31.31	31.55	31.57	32.54
32.98	33.83	33.86	34.64	35.21	36.82	37.23	38.09	38.26	38.82
38.96	38.90	42.99	43.66	44.61	45.24	47.91	54.75	69.40	98.16

#### Wave Data

The yearly maximum wave heights, in feet, observed at a given location in 50 years are given below. The data, coming from shallow water, will be used for designing a breakwater. The wave height is, by definition, a non-negative variable, which is bounded from above. In addition, we know that for shallow water an upper bound can be given, but for open sea water this bound becomes unclear.

2.91	3.74	4.09	5.88	6.42	6.93	7.21	7.92	8.26	8.79
9.17	9.50	9.62	10.00	10.14	10.28	10.45	10.77	11.65	11.65
11.82	12.27	12.68	13.28	13.46	13.88	13.98	14.32	14.38	14.46
14.86	15.03	15.30	16.07	16.23	17.36	18.68	18.72	19.44	20.09
21.06	21.13	21.53	21.80	23.15	24.75	25.45	28.13	29.95	37.19

#### Link Data

20 chain links have been tested for strength. The data is used for quality control and minimum strength characteristics are needed.

51.1	57.1	65.1	69.2	71.2	73.1	76.1	81.9	84.1	86.6
88.1	92.6	94.9	96.6	97.0	101.4	103.4	103.8	105.2	119.1

#### Fatigue Data

35 specimens of wire were tested for fatigue strength to failure with results shown below. The aim of the study is to find a design fatigue stress.

39611	44132	44209	45898	50139	54625	58970	64703	64950	66508
70208	72098	75001	80393	81868	82202	82447	89268	90021	96136
96723	101610	101833	106055	112833	119154	122366	134511	135220	136395
138378	153790	184916	216370	240316					

