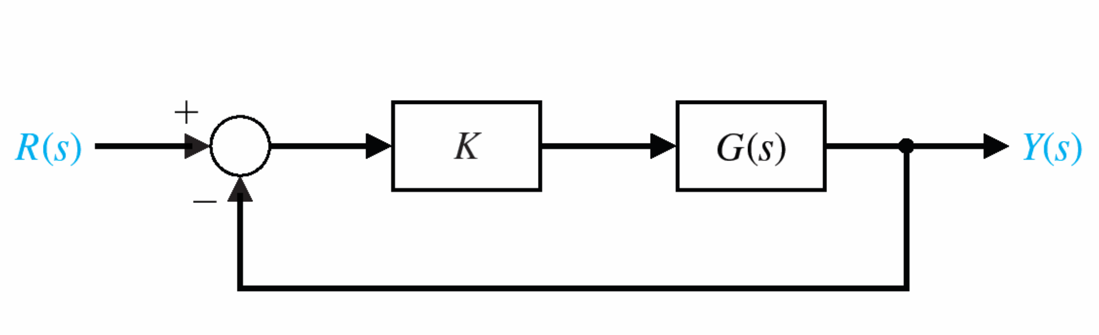
Control Systems

Part 7: Root Locus and Transient Performance

Learning objectives

- To identify the starting and finishing points of root loci
- To construct root locus
- To learn basic properties of root locus
- To understand the effect of poles and zeros on the root locus
- To interpret the root locus by predicting time domain responses

What is Root Locus ?



The characteristic equation of the closed-loop system is

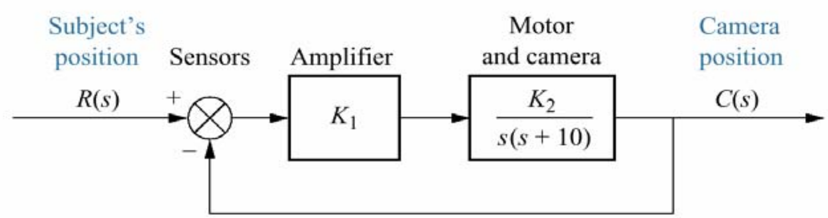
$$1 + K\,G(s) = 0$$

The root locus is essentially the trajectories of roots of the characteristic equation as the parameter *K* is varied from 0 to infinity.

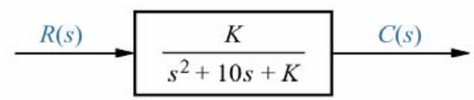
A simple example

A camera control system:

(a)



(b)



where $K = K_1K_2$

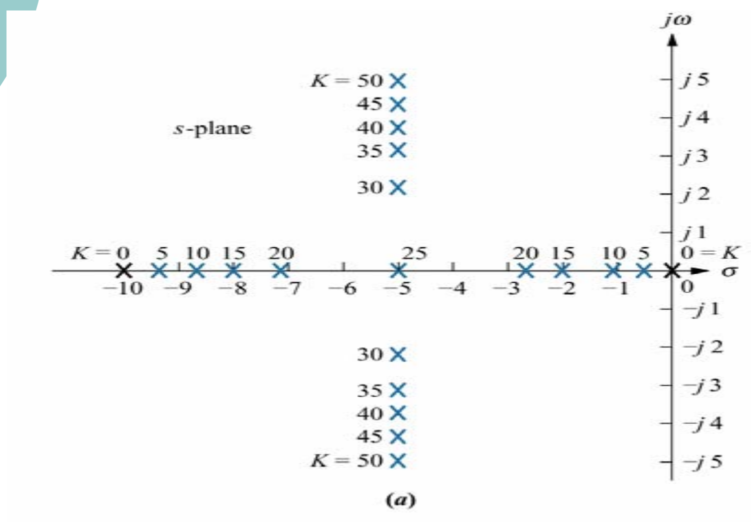
(c)

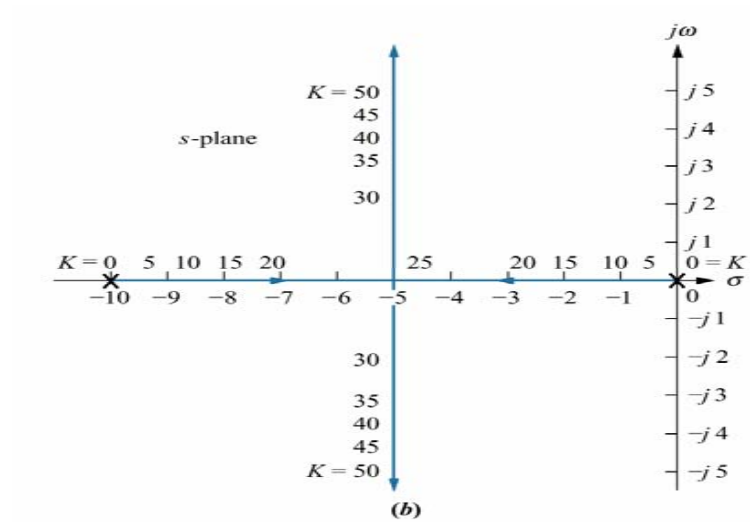
How the dynamics of the camera changes as K is varied ?

A simple example (cont.) : pole locations

K	Pole 1	Pole 2
0	-10	0
5	-9.47	-0.53
10	-8.87	-1.13
15	-8.16	-1.84
20	-7.24	-2.76
25	-5	-5
30	$-5 + j2.24$	$-5 - j2.24$
35	$-5 + j3.16$	$-5 - j3.16$
40	$-5 + j3.87$	$-5 - j3.87$
45	$-5 + j4.47$	$-5 - j4.47$
50	$-5 + j5$	$-5 - j5$

A simple example (cont.) : Root Locus





(a) Pole plots from the table.

(b) Root locus.

Main properties of root locus

- Locus always starts from the open-loop poles
- Locus always ends at the open-loop zeros or infinity
- The number of branches of loci is equal to the order of the system, i.e. the number of open-loop poles
- Root locus is always symmetric with respect to real axis
- Locus goes to infinity by following certain asymptotes
- The section of the real axis is a part of locus if and only if the sum of the number of poles and zeros to its right is odd
- Routh-Hurwitz criteria can be used to determine the gain value at which the locus goes across the imaginary axis.

Location and angles of asymptotes

The intersection of the asymptotes with the real axis can be determined by:

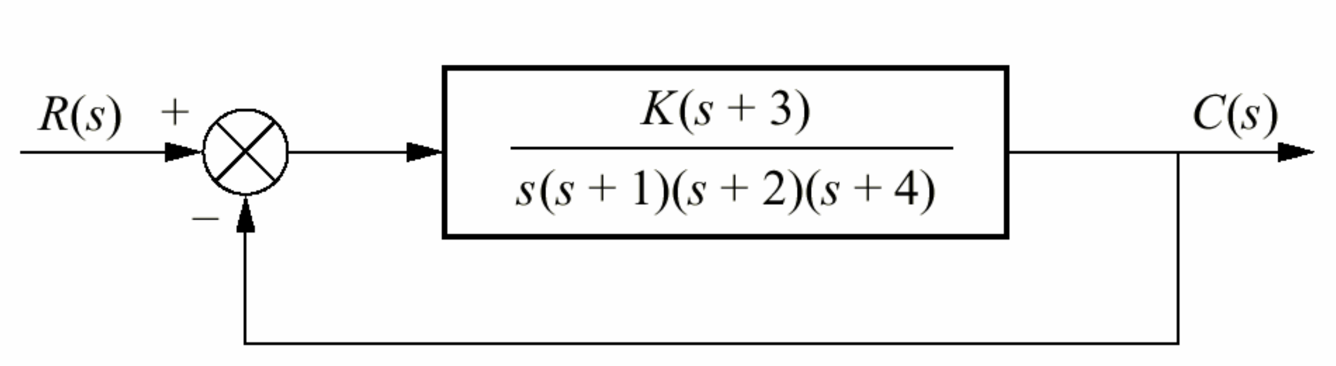
$$\sigma_{a} = \frac{\sum \text{finite} \quad \text{poles} - \sum \text{finite} \quad \text{zeros}}{\#\text{finite} \quad \text{poles} - \#\text{finite} \quad \text{zero}}$$

The angles of the asymptotes are

$$\theta_{a} = \frac{(2k+1)\pi}{\#\text{finite} \quad \text{poles} - \#\text{finite} \quad \text{zeros}}$$

Example of root locus construction

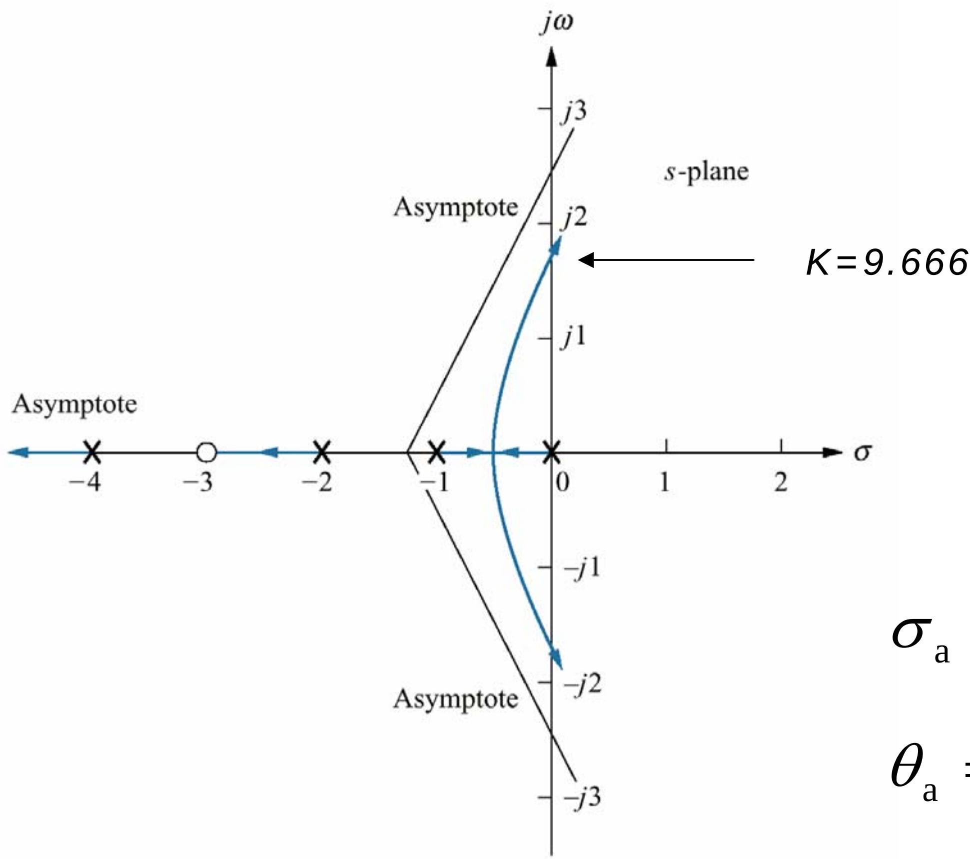
Construct a root locus for the following system



Basically, we want to solve the roots of the following characteristic equation as a function of the parameter K.

$$s(s+1)(s+2)(s+4)+K(s+3)=0$$

Example of root locus construction (Cont.)



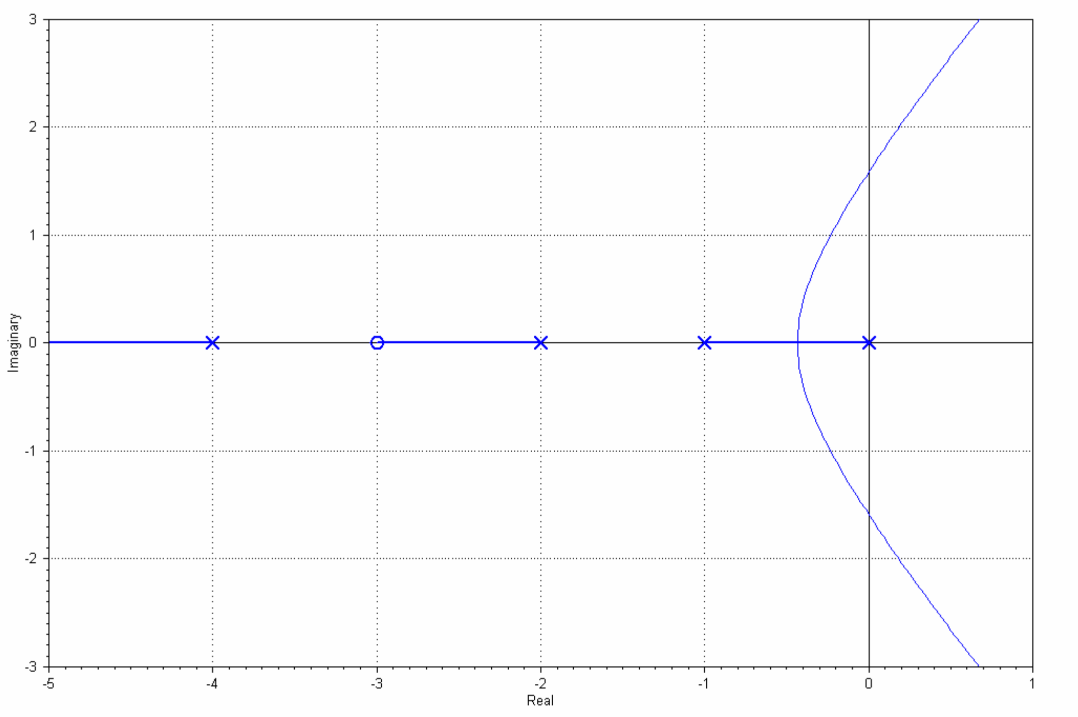
$$\sigma_a = -\frac{4}{3}$$

$$\theta_a = \pi/3, \qquad \pi, \qquad 5\pi/3$$

Construction of root locus using CC

cc> g=(s+3)/s/(s+1)/(s+2)/(s+3)

cc> rootlocus

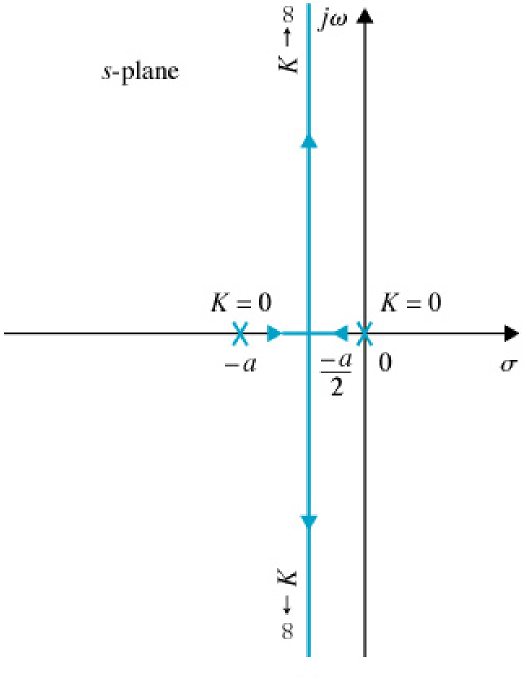


Effect of adding poles (A)

Adding a pole to open-loop transfer function has the effect of pushing the root loci toward the right half of s-plane.

Let's consider:

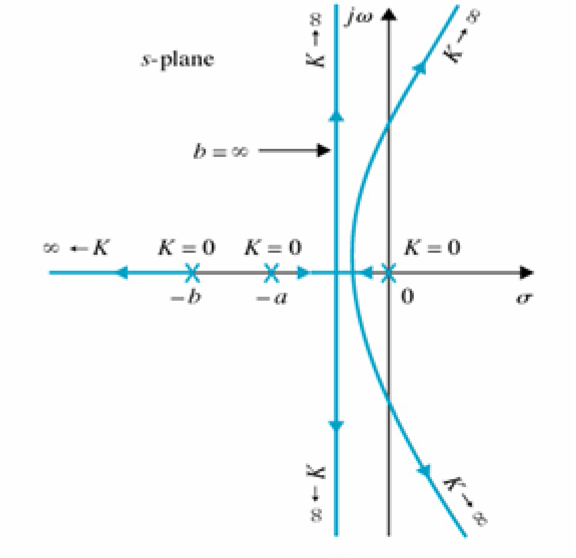
$$KG(s) = \frac{K}{s(s+a)} \qquad a > 0$$



Effect of adding poles (B)

Adding one pole to this system, we have

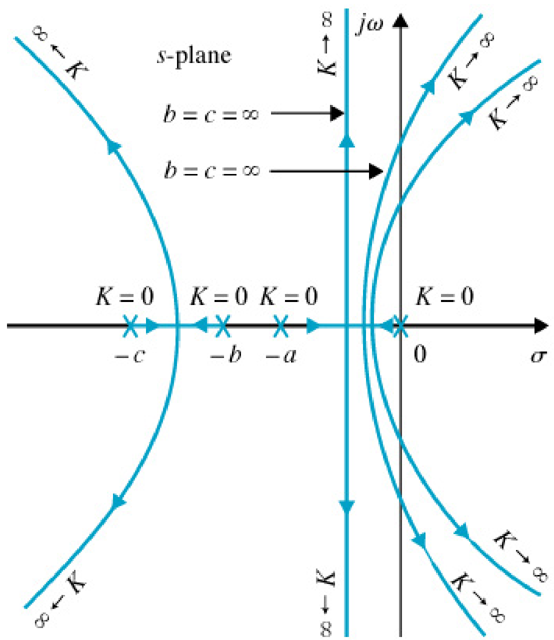
$$KG(s) = \frac{K}{s(s+a)(s+b)} \qquad b > a > 0$$



Effect of adding poles (C)

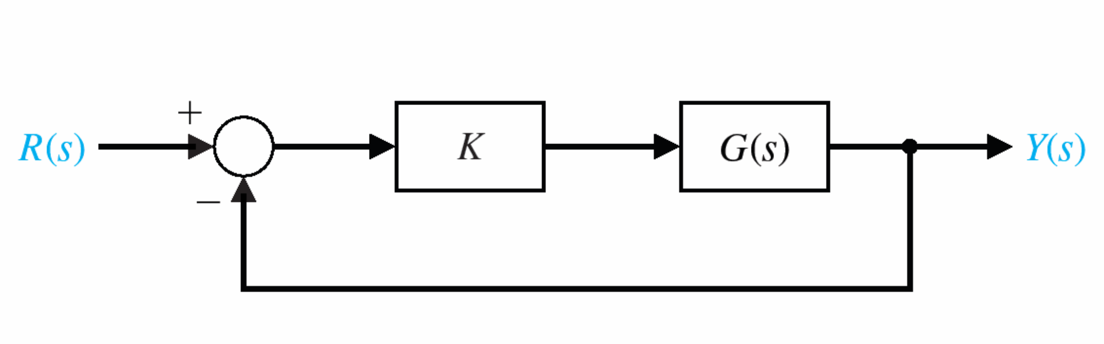
If we add another pole, i.e.

$$KG(s) = \frac{K}{s(s+a)(s+b)(s+c)} \qquad c > b > a > 0$$



(c)

Let's consider the following system:

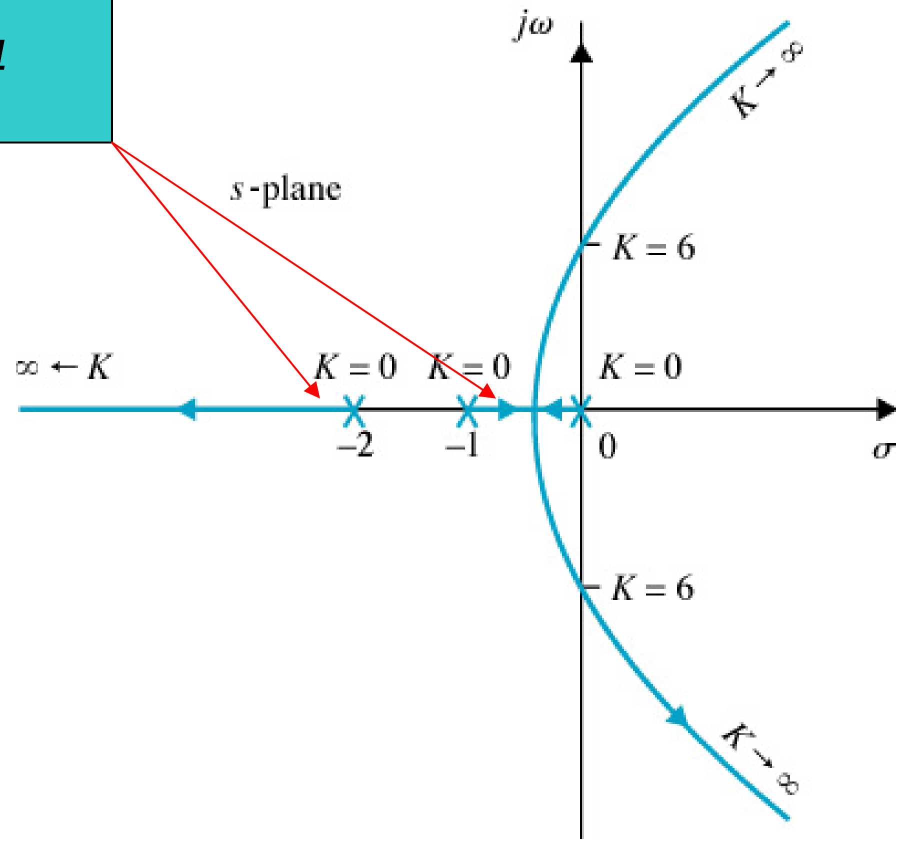


where

$$G(s) = \frac{1}{s(s+1)(s+2)}$$

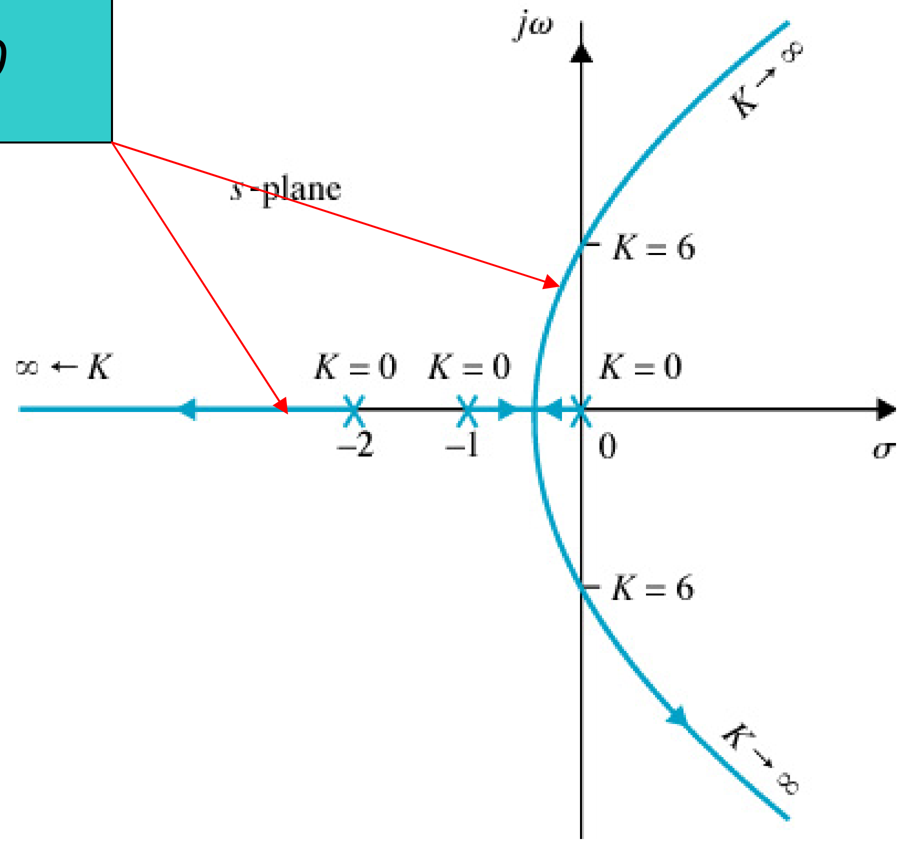
Root locus of this example (low gain)

K= 0.4



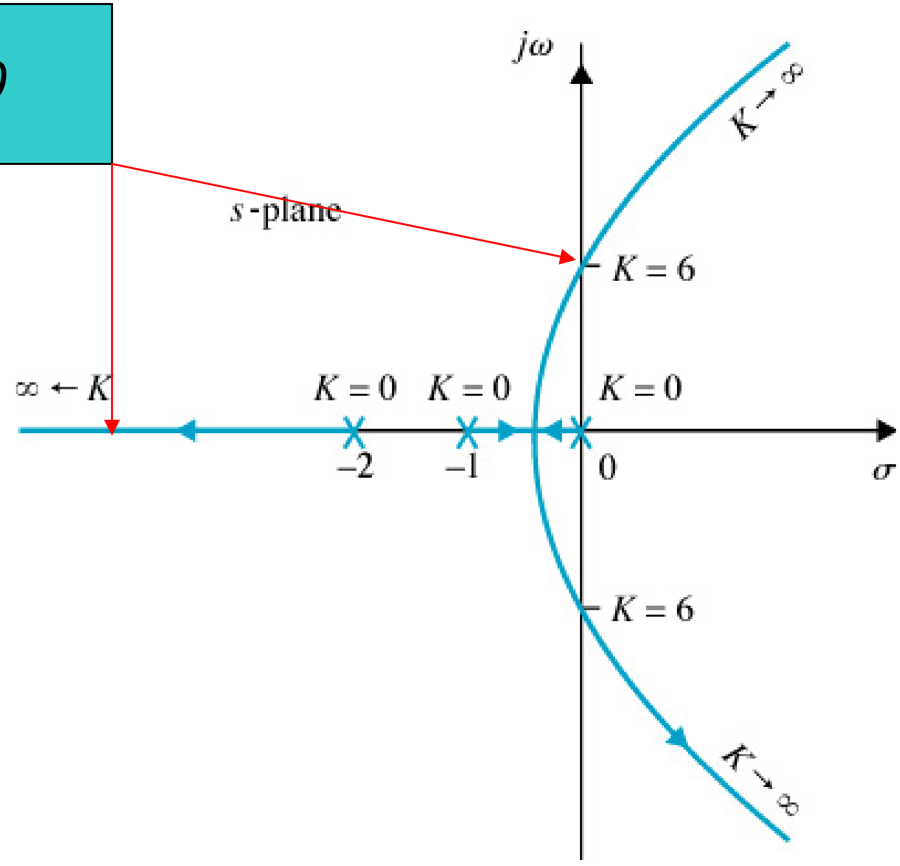
Root locus of this example (media gain)

K= 2.0



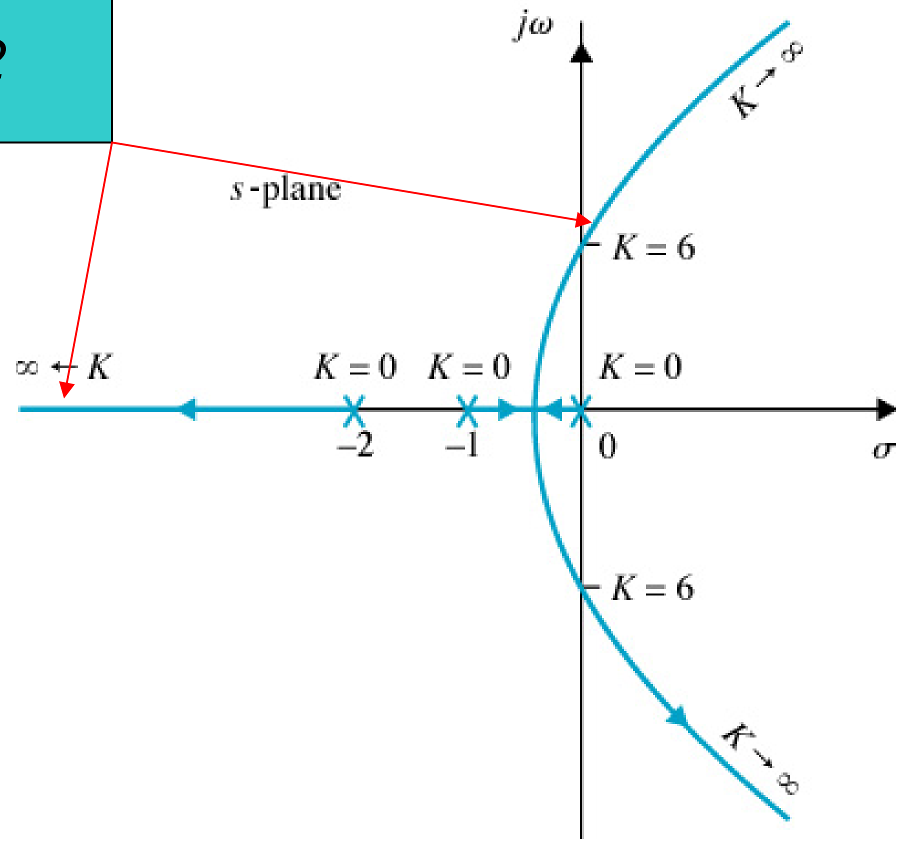
Root locus of this example (critical gain)

K= 6.0

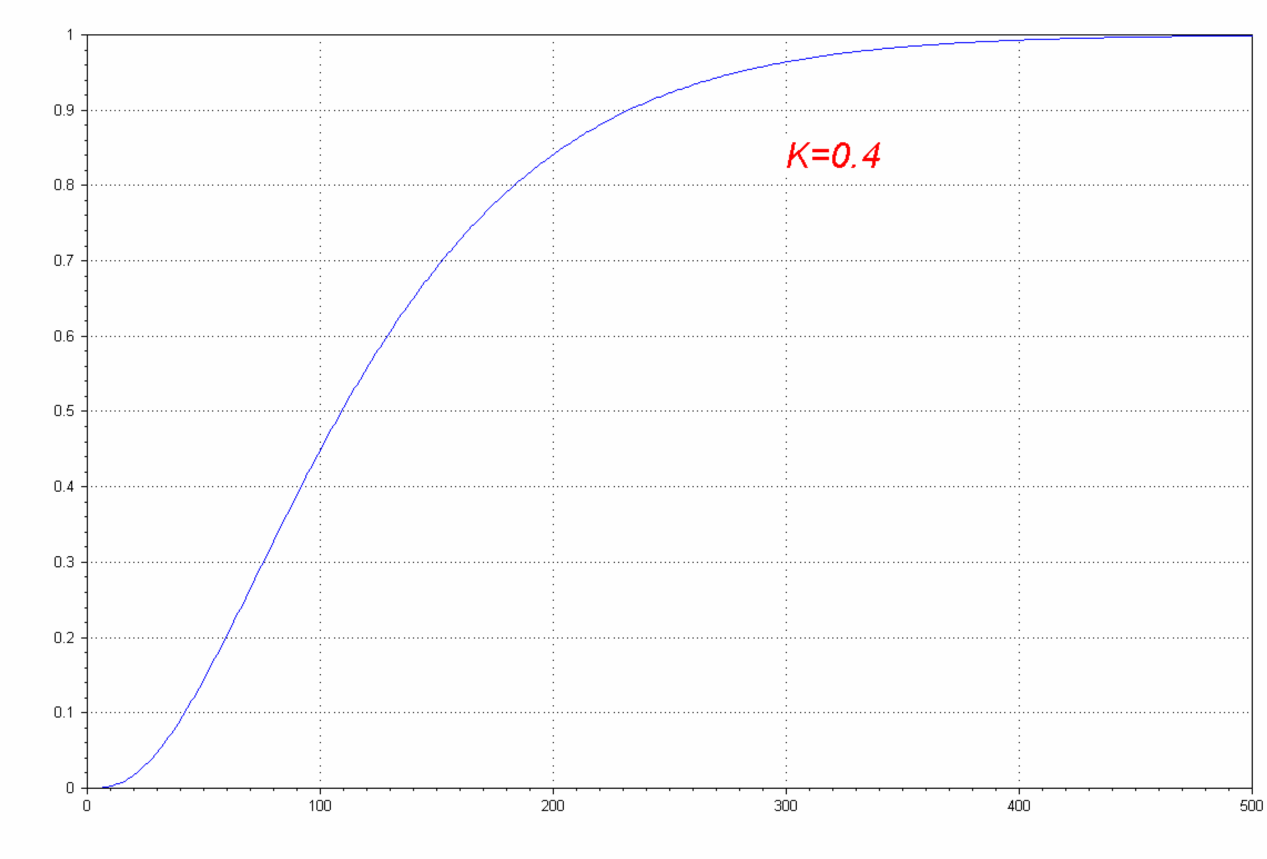


Root locus of this example (high gain)

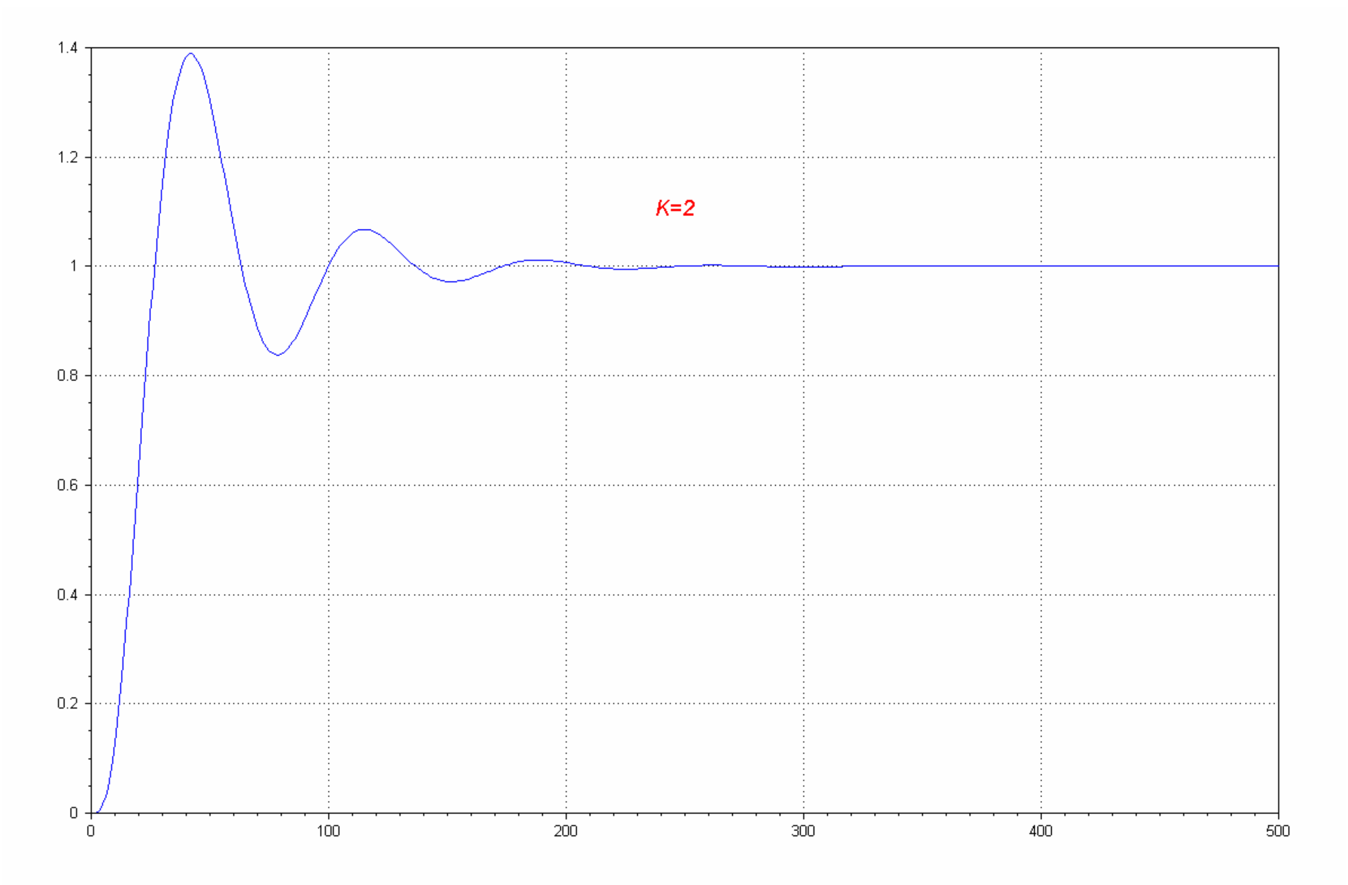
K= 6.2



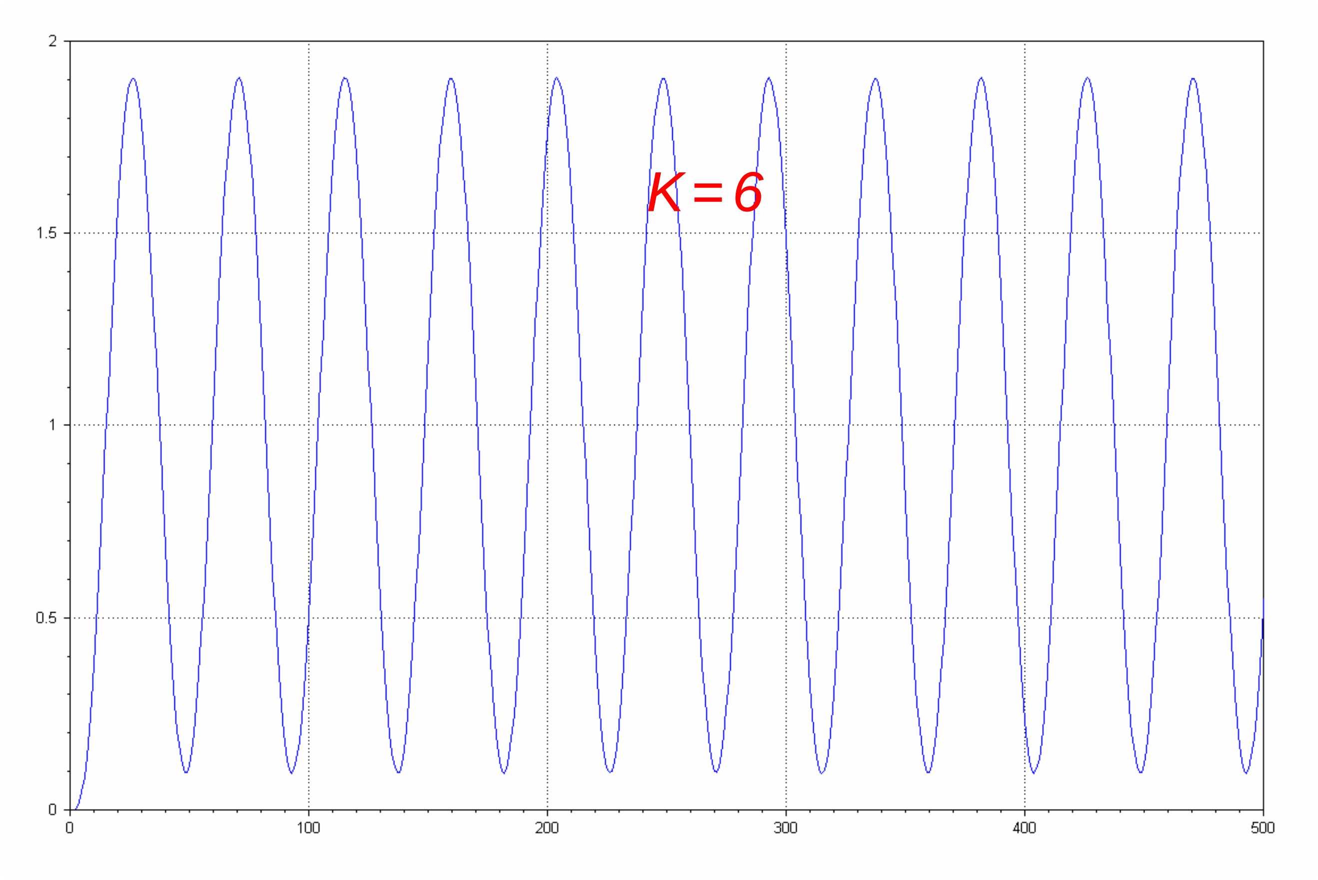
Time domain step response (low gain)



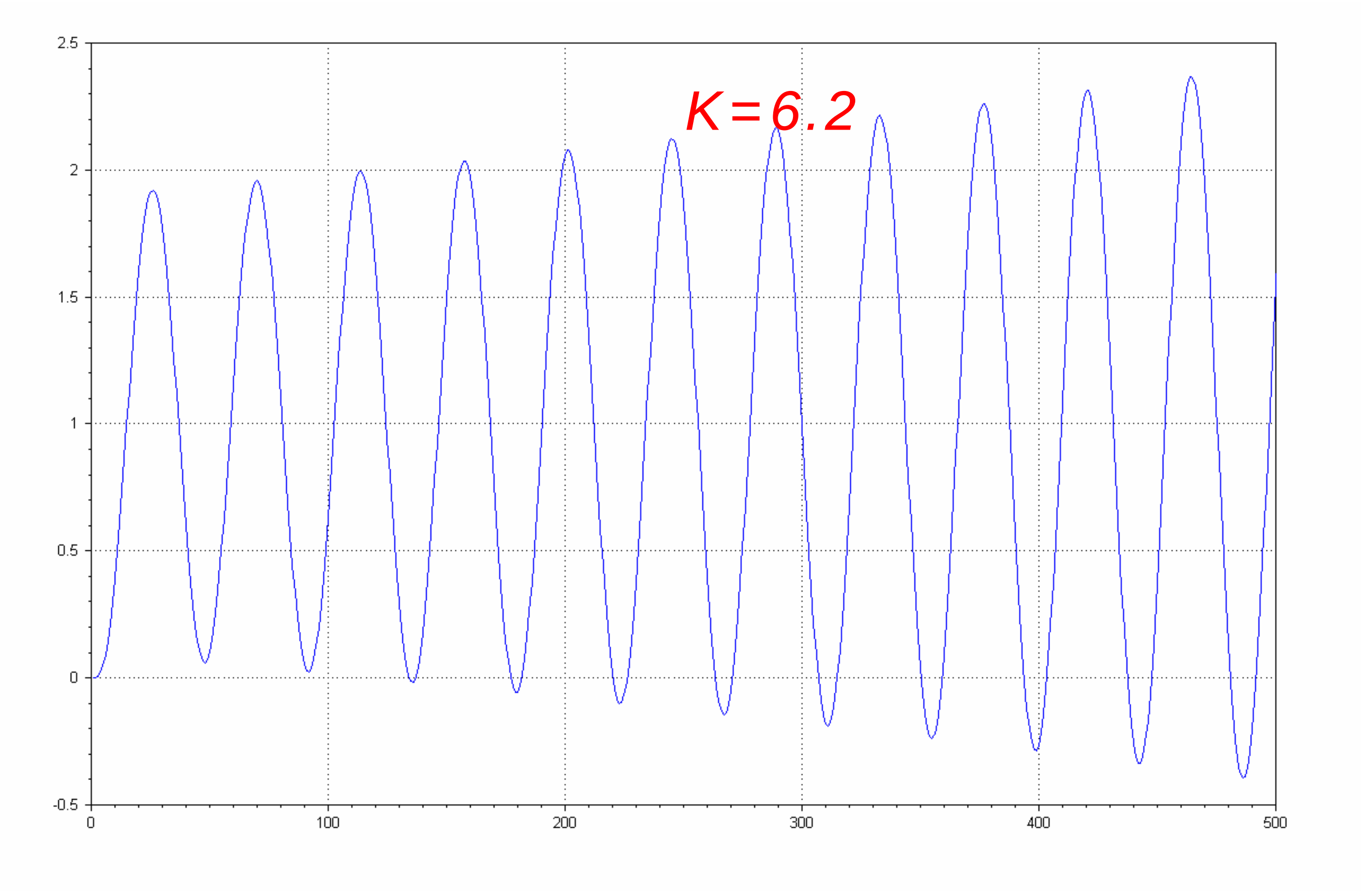
Time domain step response (media gain)



Time domain step response (critical gain)



Time domain step response (high gain)



Control system design using root locus method

- One can modify the dynamic behavior of a closed-loop system by introducing extra poles and zeros to shape up the system characteristics.
- One can also select appropriate gain values to achieve the desired transient response.
- Root locus technique can also be used to study the effects of the multiple system parameters by varying one parameter at a time.