



# Control Systems

---

## Part 6: Classical PID Control

## Learning objectives

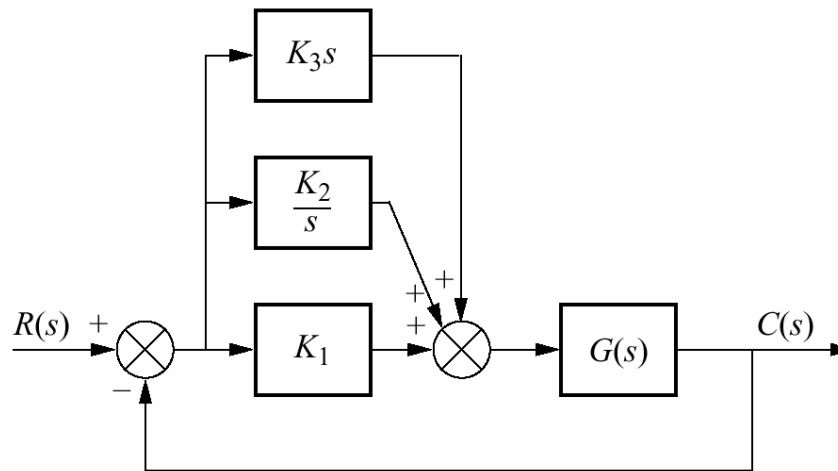
---

- To state the definition of PID controllers
- To learn different forms of controller structures
- To learn different tuning techniques for PID controllers
- To understand the effect of each controller parameter on the system responses.

## PID controllers

PID stands for:

- P** (*Proportional*)
- I** (*Integral*)
- D** (*Derivative*)





## Historical note

---



The first application of PID controller was in 1922 by Minorsky on ship steering.

Minorsky (1922) "Directional stability of automatically steered bodies", *J. Am. Soc. Naval Eng.*, 34, p.284.

This was the first mathematical treatment of the type of controller that is now used to control almost all industrial processes.

---

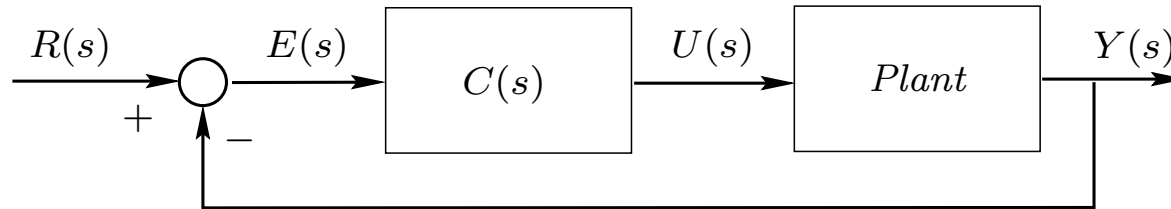
Despite the abundance of sophisticated tools, including advanced controller design techniques, PID controllers are still the most widely used controller structure in modern industry, controlling more than 95% of closed-loop industrial processes.

Different PID controllers differ in the way how their parameters are tuned, manually, or automatically.

Most of the DCS systems have built-in routines to perform auto-tuning of PID controllers based on the loop characteristics. They are often called: auto-tuners.

## PID structure

Consider the simple SISO control loop shown below



$C(s)$  is the controller

Standard PID controllers have the following structures:

*Proportional only:*  $C_P(s) = K_p$

*Proportional plus Integral:*  $C_{PI}(s) = K_p \left( 1 + \frac{1}{T_r s} \right)$

*Proportional plus derivative:*  $C_{PD}(s) = K_p \left( 1 + \frac{T_d s}{\tau_D s + 1} \right)$

*Proportional, integral and derivative:*  $C_{PID}(s) = K_p \left( 1 + \frac{1}{T_r s} + \frac{T_d s}{\tau_D s + 1} \right)$

## PID structures

---

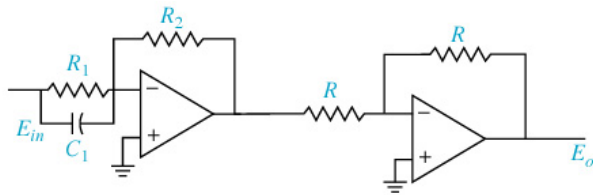
An alternative series form is:

$$C_{series}(s) = K_s \left( 1 + \frac{I_s}{s} \right) \left( 1 + \frac{D_s s}{\gamma_s D_s s + 1} \right)$$

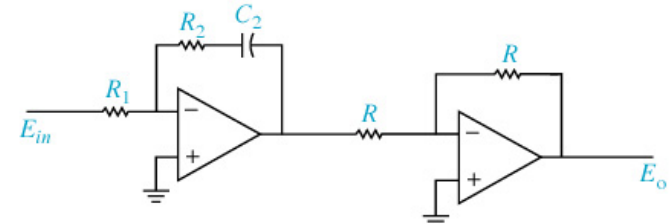
Yet another alternative form is the, so called, parallel form:

$$C_{parallel}(s) = K_p + \frac{I_p}{s} + \frac{D_p s}{\gamma_p D_p s + 1}$$

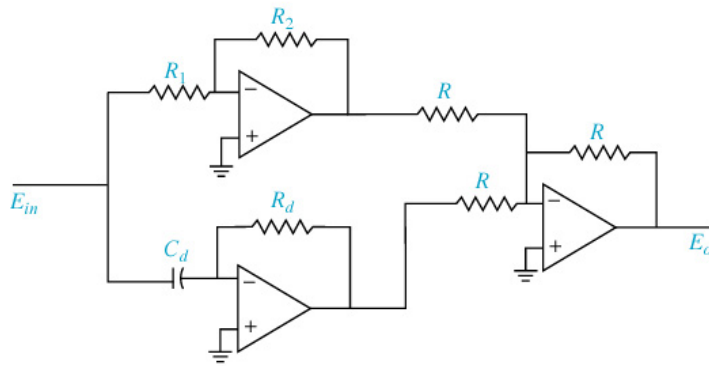
# Analog electronic implementation of controllers



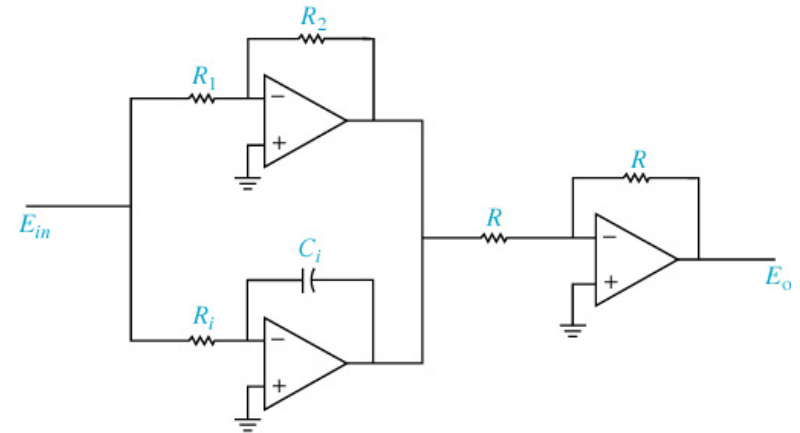
(a)



(a)



(b)



(b)

PD controllers

PI controllers

## Tuning techniques for PID controllers

---

Because of their widespread use in practice, we present below several methods for tuning PID controllers. Actually these methods are quite old and date back to the 1950's. Nonetheless, they remain in widespread use today.

- *Ziegler-Nichols Oscillation Method*
- *Ziegler-Nichols Reaction Curve Method*
- *Cohen-Coon Reaction Curve Method*

## Ziegler-Nichols (Z-N) Oscillation Method

---

This procedure is only valid for open loop stable plants and it is carried out through the following steps

- Set the true plant under proportional control, with a very small gain.
- Increase the gain until the loop starts oscillating. Note that linear oscillation is required and that it should be detected at the controller output.

## Ziegler-Nichols (Z-N) Oscillation Method

- Record the controller critical gain  $K_p = K_c$  and the oscillation period of the controller output,  $P_c$ .
- Adjust the controller parameters according to the following table.

	$K_p$	$T_r$	$T_d$
<b>P</b>	$0.50K_c$		
<b>PI</b>	$0.45K_c$	$\frac{P_c}{1.2}$	
<b>PID</b>	$0.60K_c$	$0.5P_c$	$\frac{P_c}{8}$

## PID Tuning Example

---

Consider a plant with a model given by

$$G_o(s) = \frac{1}{(s + 1)^3}$$

Find the parameters of a PID controller using the Z-N oscillation method. Obtain a graph of the response to a unit step input reference and to a unit step input disturbance.

Applying the procedure outlined before, we have

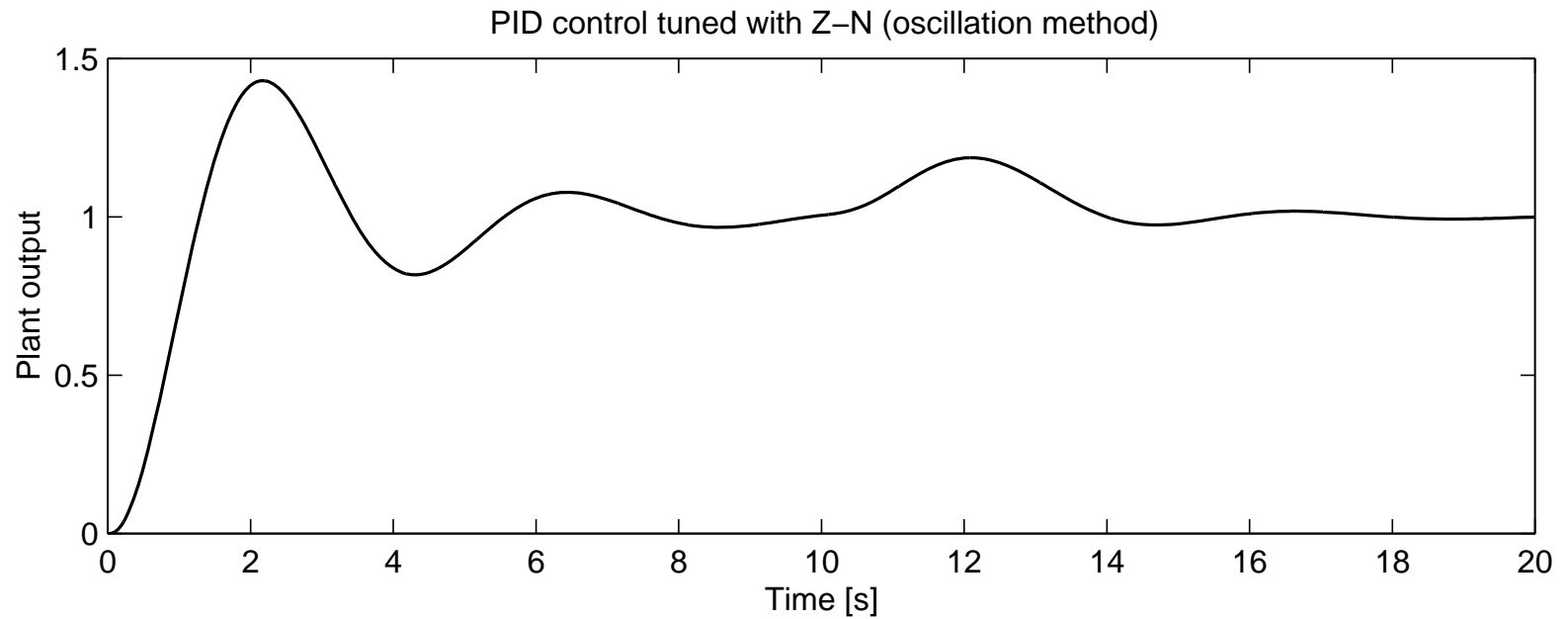
$$K_c = 8 \quad \text{and} \quad \omega_c = \sqrt{3}.$$

Hence, from the Table, it follows that

$$K_p = 0.6 * K_c = 4.8; \quad T_r = 0.5 * P_c \approx 1.81; \quad T_d = 0.125 * P_c \approx 0.45$$

The closed loop response to a unit step in the reference at  $t = 0$  and a unit step disturbance at  $t = 10$  are shown next.

## Step response of PID controlled system



The PID Controller parameters can also be determined by performing a open-loop step test detailed as follows:

1. With the plant in open loop, take the plant manually to a normal operating point. Say that the plant output settles at  $y(t) = y_0$  for a constant plant input  $u(t) = u_0$ .
2. At an initial time,  $t_0$ , apply a step change to the plant input, from  $u_0$  to  $u_\infty$  (*this should be in the range of 10 to 20% of full scale*).

## Reaction Curve Based Methods (Cont)

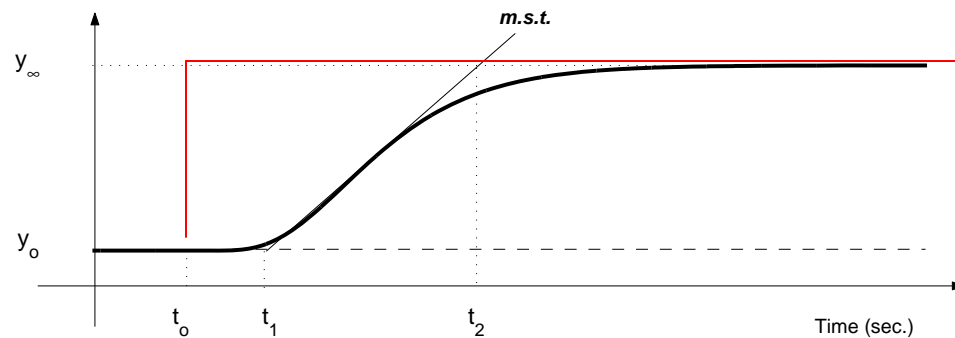
---

- Record the plant output until it settles to the new operating point. This curve is known as the *process reaction curve*.
- Compute the following parameters

$$K_o = \frac{y_\infty - y_o}{u_\infty - u_o}; \quad \tau_o = t_1 - t_o; \quad \nu_o = t_2 - t_1$$

## Reaction Curve Based Methods (Cont)

Based on the parameters recorded in the reaction curve, the parameters of the PID controller can be calculated as shown in the table on next page.



where m.s.t. stands for *maximum slope tangent*.

## Reaction Curve Based Methods (Cont)

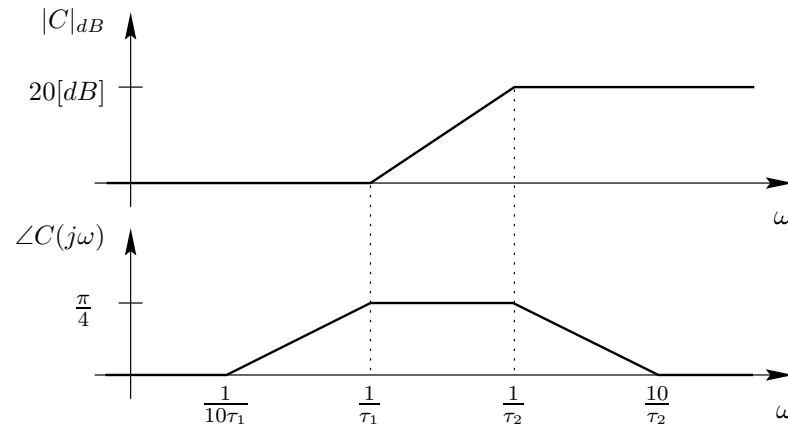
	$K_p$	$T_r$	$T_d$
<b>P</b>	$\frac{\nu_o}{K_o \tau_o}$		
<b>PI</b>	$\frac{0.9\nu_o}{K_o \tau_o}$	$3\tau_o$	
<b>PID</b>	$\frac{1.2\nu_o}{K_o \tau_o}$	$2\tau_o$	$0.5\tau_o$

Closely related to PID control is the idea of lead-lag compensation. The transfer function of these compensators is of the form:

$$C(s) = \frac{\tau_1 s + 1}{\tau_2 s + 1}$$

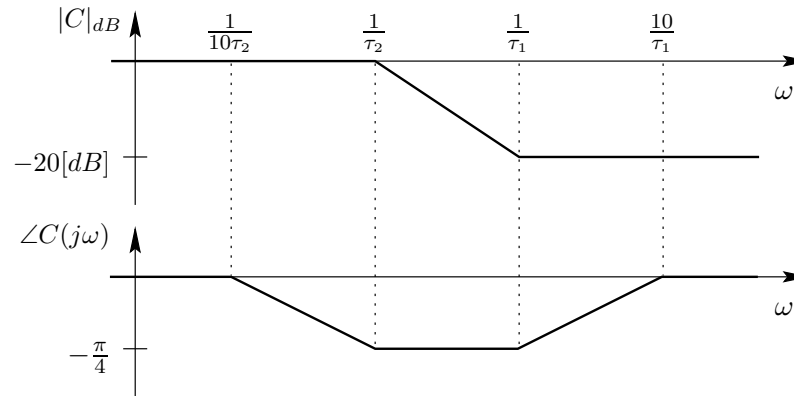
If  $\tau_1 > \tau_2$ , then this is a *lead network* and when  $\tau_1 < \tau_2$ , this is a *lag network*.

## Frequency response of a lead-lag compensator



We see from the previous slide that the lead-lag compensator gives phase advance at  $\omega = 1/\tau_1$  without an increase in gain. Thus it plays a role similar to derivative action in PID.

## Frequency response of a lag-lead compensator



We see from the previous slide that the lag-lead compensator gives low frequency gain increase. Thus it plays a role similar to integral action in PID.

## Summary

---

- PI and PID controllers are widely used in industrial control.
- From a modern perspective, a PID controller is simply a controller of (up to second order) containing an integrator. Historically, however, PID controllers were tuned in terms of their **P**, **I** and **D** terms.
- It has been empirically found that the PID structure often has sufficient flexibility to yield excellent results in many applications.

## Summary

---

- The basic term is the proportional term, **P**, which causes a corrective control actuation proportional to the error.
- The integral term, **I** gives a correction proportional to the integral of the error. This has the positive feature of ultimately ensuring that sufficient control effort is applied to reduce the tracking error to zero. However, integral action tends to have a destabilizing effect due to the increased phase delay.

## Summary

---

- The derivative term, **D**, provides a predictive capability yielding a control action proportional to the rate of change of the error. This tends to have a stabilizing effect but often leads to large control movements.
- Various empirical tuning methods can be used to determine the PID parameters for a given application. They should be considered as a first guess in a search procedure.

## Summary

---

- Attention should also be paid to the PID structure.
- Systematic model-based procedures for PID controllers will be covered in later chapters.
- A controller structure that is closely related to PID is a lead-lag network. The lead component acts like **D** and the lag acts like **I**.