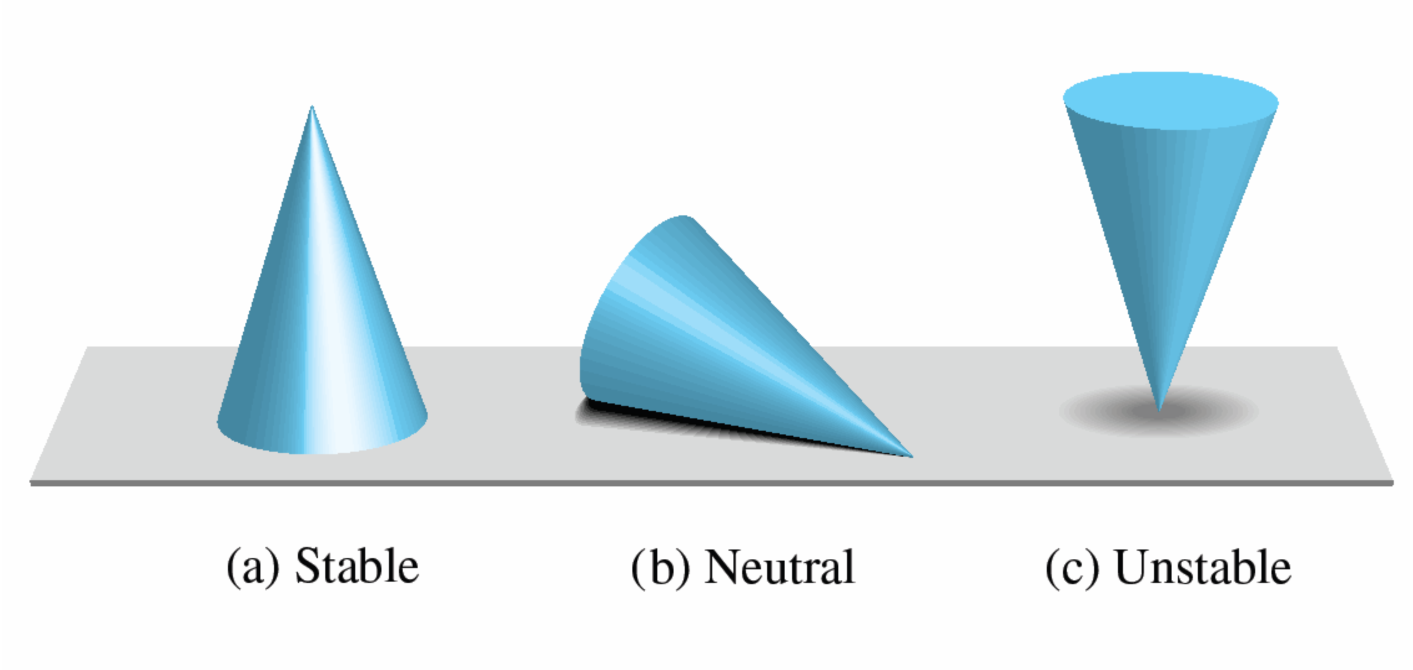
Control Systems

Part 4: Transient and Steady-state Responses

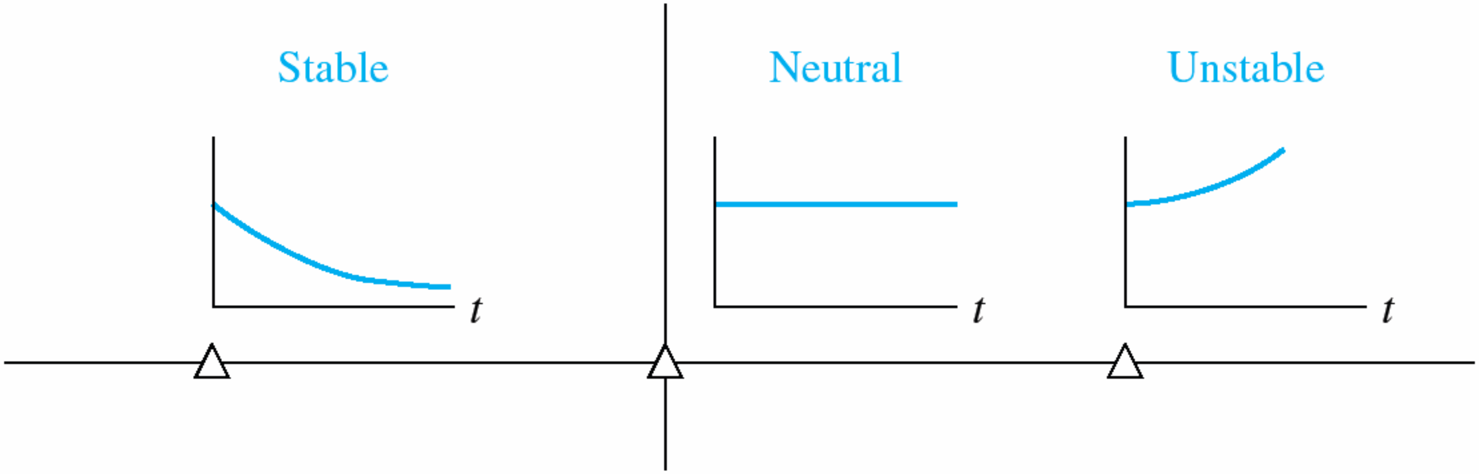
Learning objectives

- To state the concept of system stability
- To check the stability of closed-loop system from transfer function
- To examine different steady-state performance measures in time domain
- To understand the trade-offs between steady-state and transient performance.

Basic concept of stability



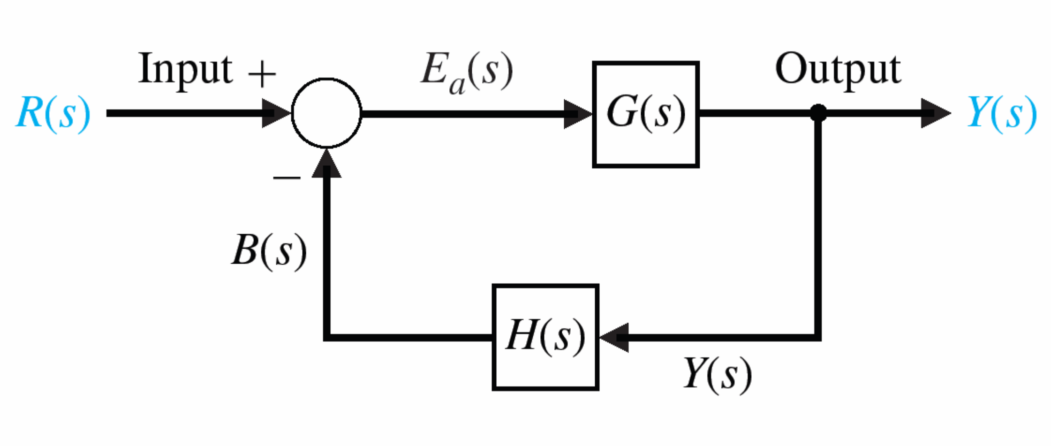
Shape of time domain response



Consequences of instability

(a)

Characteristic equation of the system

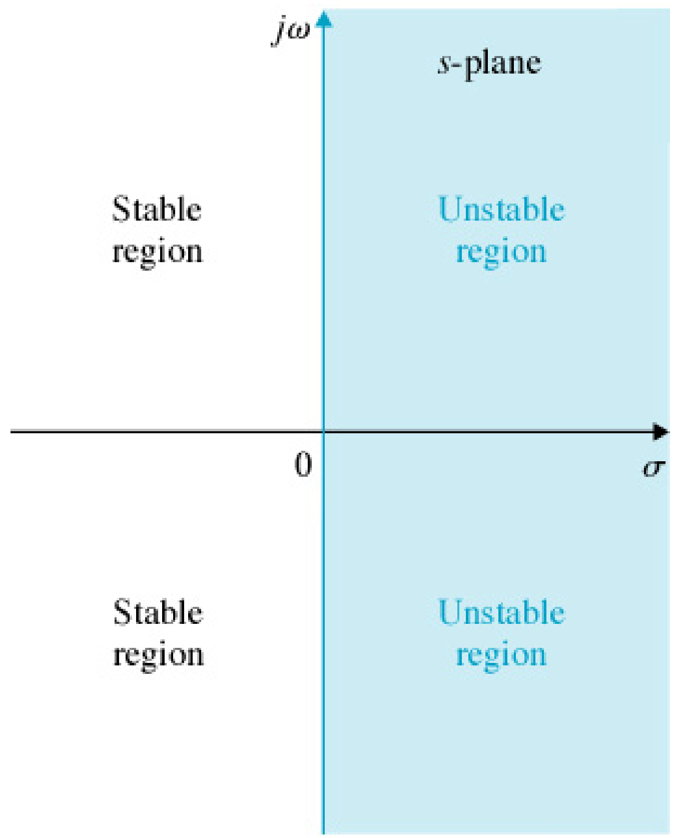


$$\frac{Y(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

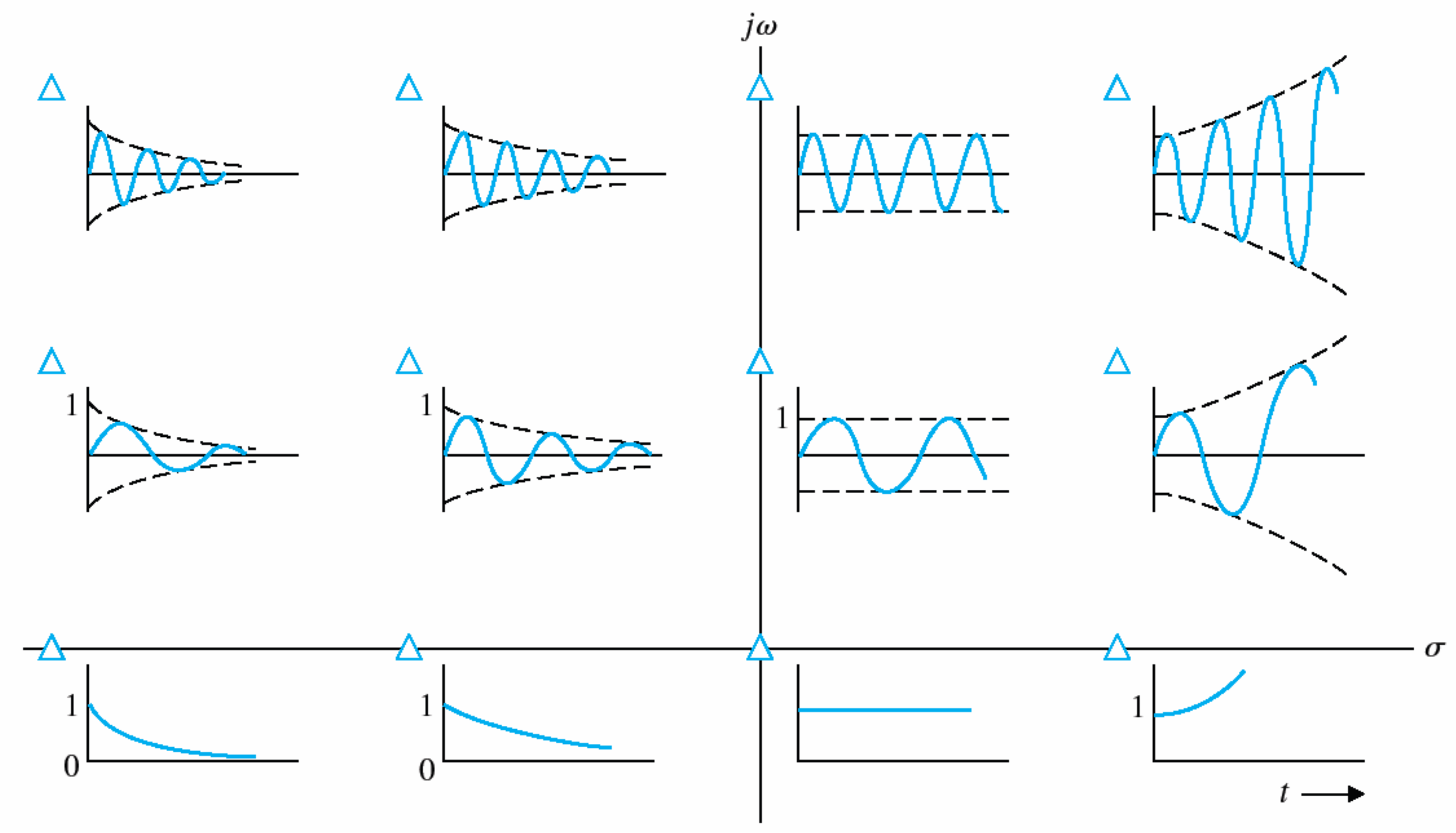
$$1 + G(s)H(s) = 0$$

Characteristic Equation

Stability of the system and roots of characteristic equations



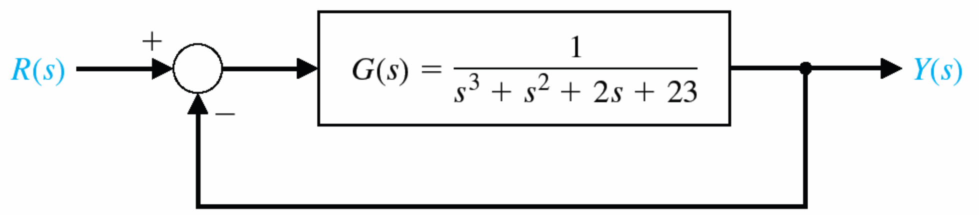
Stability of the system and roots of characteristic equations



Routh-Hurwitz Criterion

It is a technique that one can use to check the stability of the system from the characteristic equation without solving it.

Let's consider



Determine the closed-loop stability of this system

Routh-Hurwitz Criterion example (Cont)

The characteristic equation of the system is:

$$1 + G(s) = 0$$

$$s^3 + s^2 + 2s + 24 = 0$$

The Routh-Hurwitz table can be formulated as follows:

Routh-Hurwitz Criterion example (Cont)

s^3	1	2
s^2	1	24
s^1	-22	0
s^0	24	0

1st sign change

2nd sign change

Routh-Hurwitz Criterion example (Cont)

Since there are two changes in the sign of the first column of the Routh-Hurwitz table, there are two unstable poles in the closed-loop system.

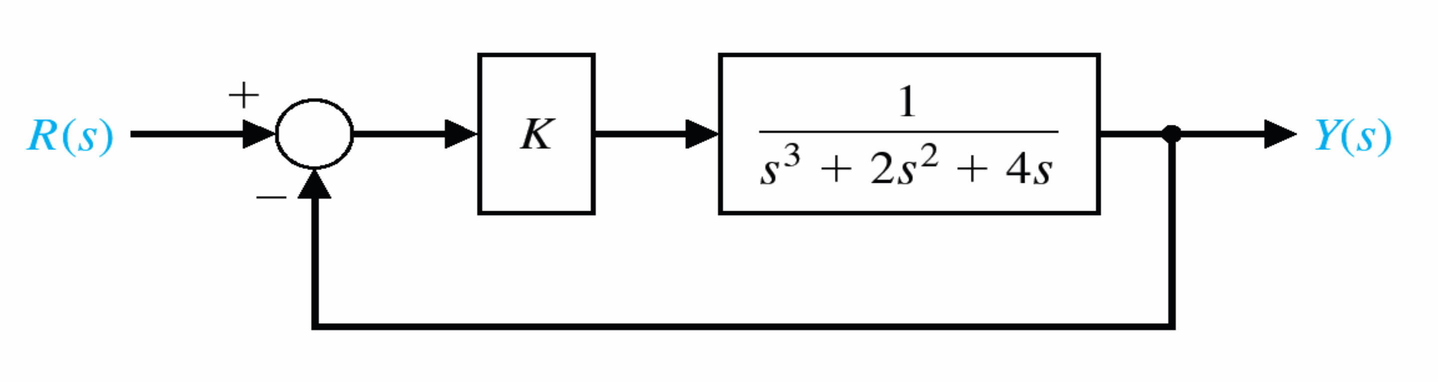
In fact, the roots are:

-3.0000
1.0000 + 2.6458i
1.0000 - 2.6458i

Unstable poles

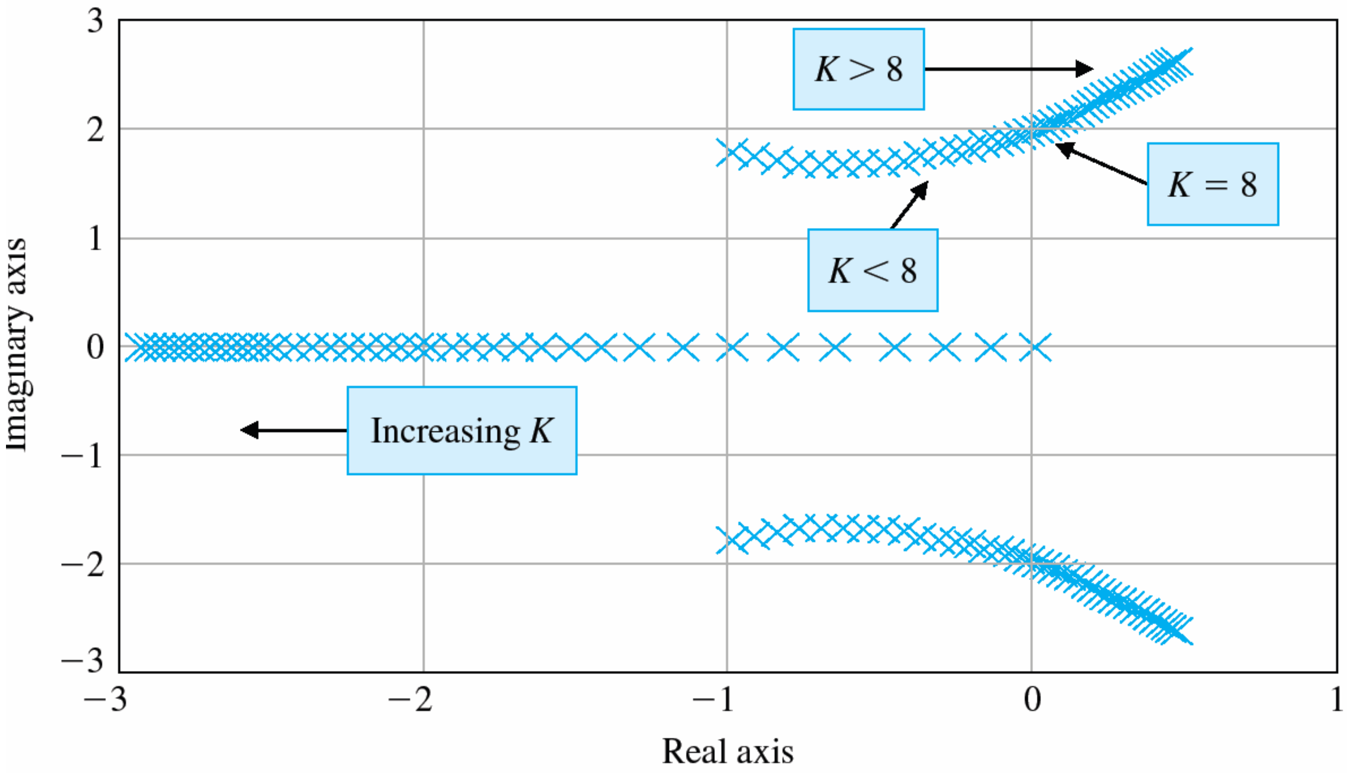
Stability as a function of system parameters

Let's consider



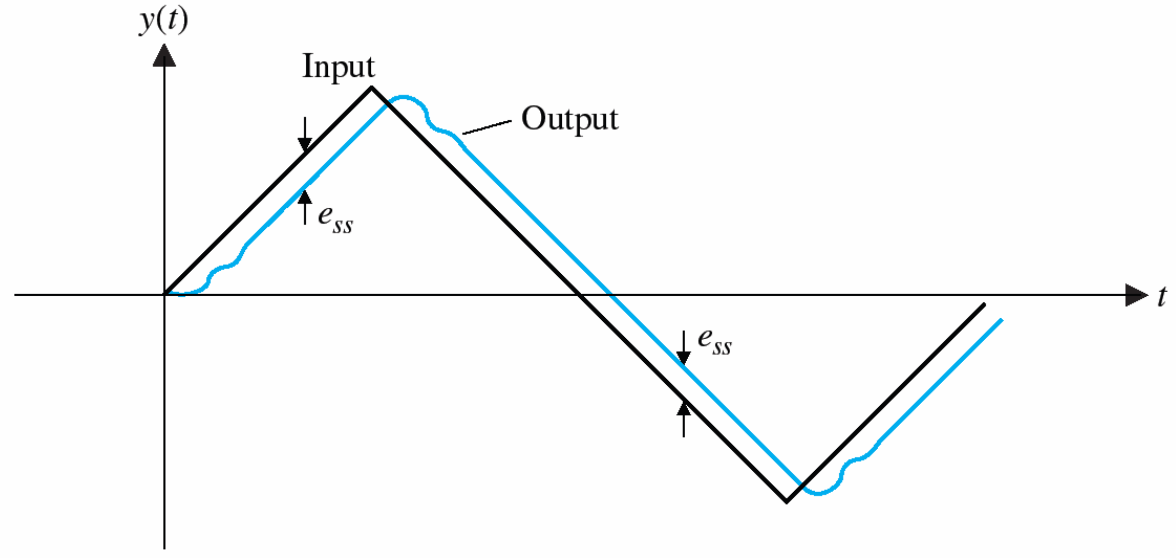
Determine the stability of the system as a function of the parameter K.

Stability as a function of system parameters (Cont)

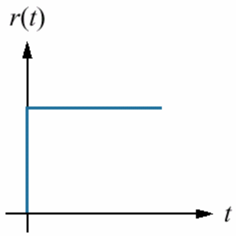
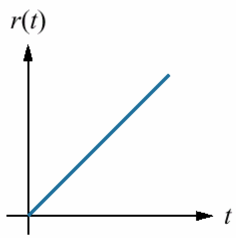
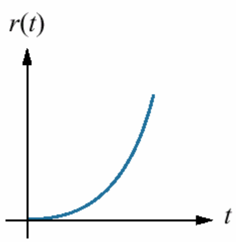


Steady-state behavior of the system

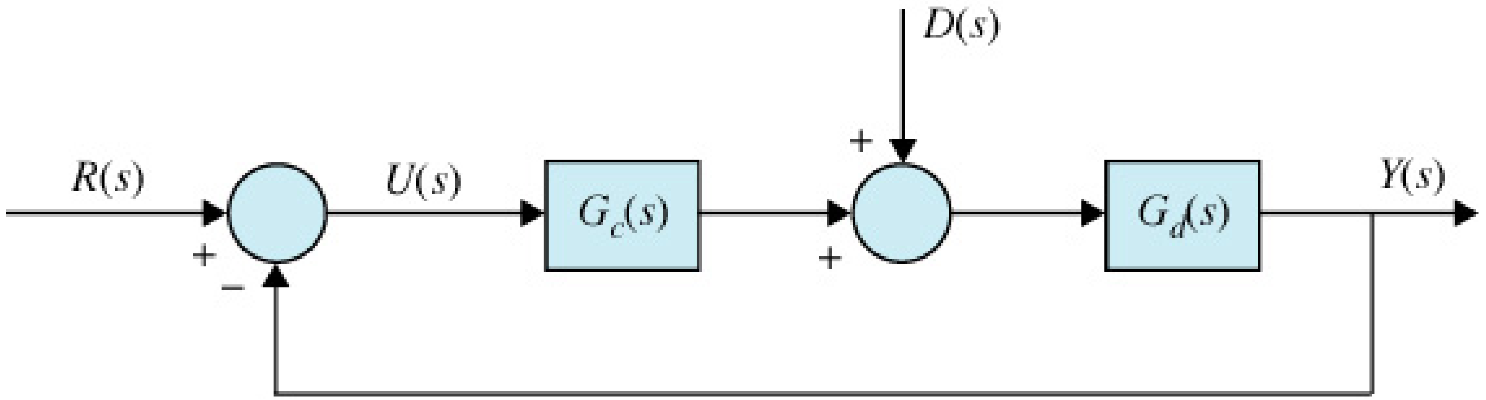
Steady-state accuracy of the control system is also very important.



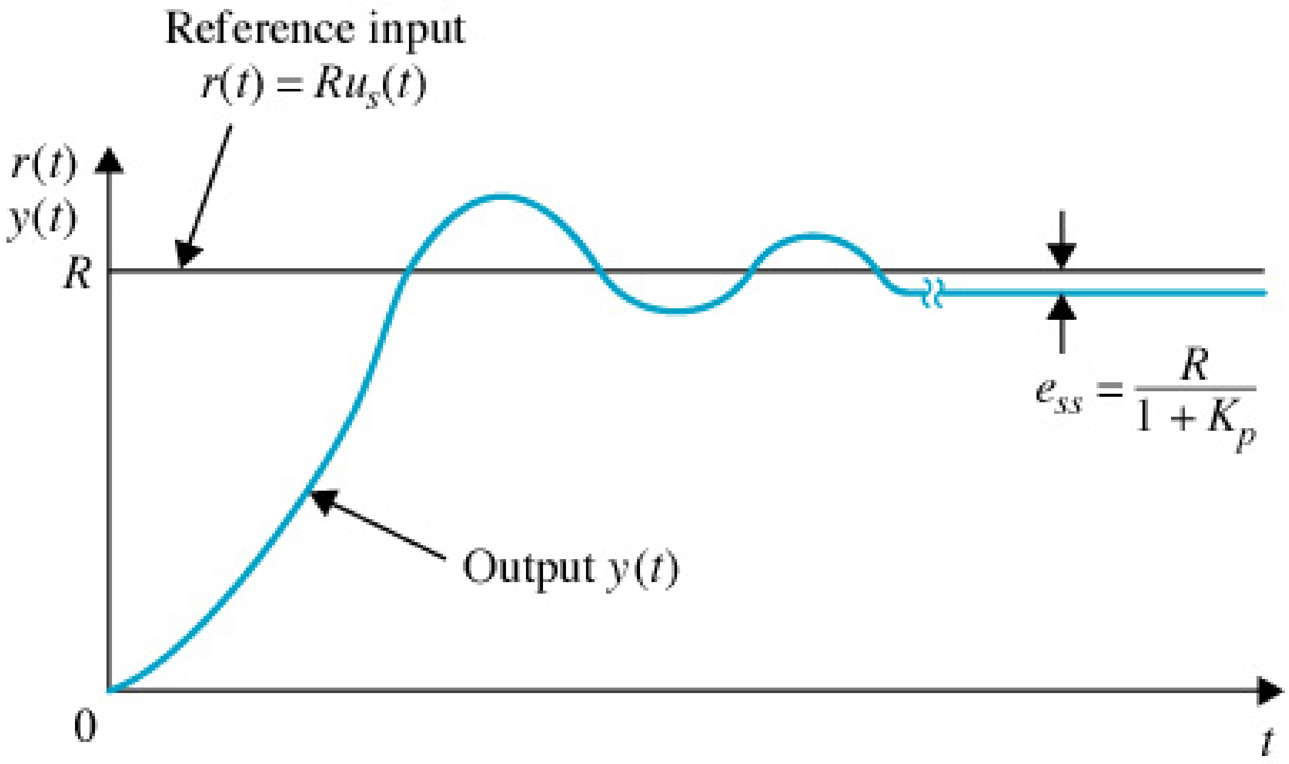
Standard test inputs

Waveform	Name	Physical interpretation	Time function	Laplace transform
	Step	Constant position	1	$\frac{1}{s}$
	Ramp	Constant velocity	t	$\frac{1}{s^2}$
	Parabola	Constant acceleration	$\frac{1}{2}t^2$	$\frac{1}{s^3}$

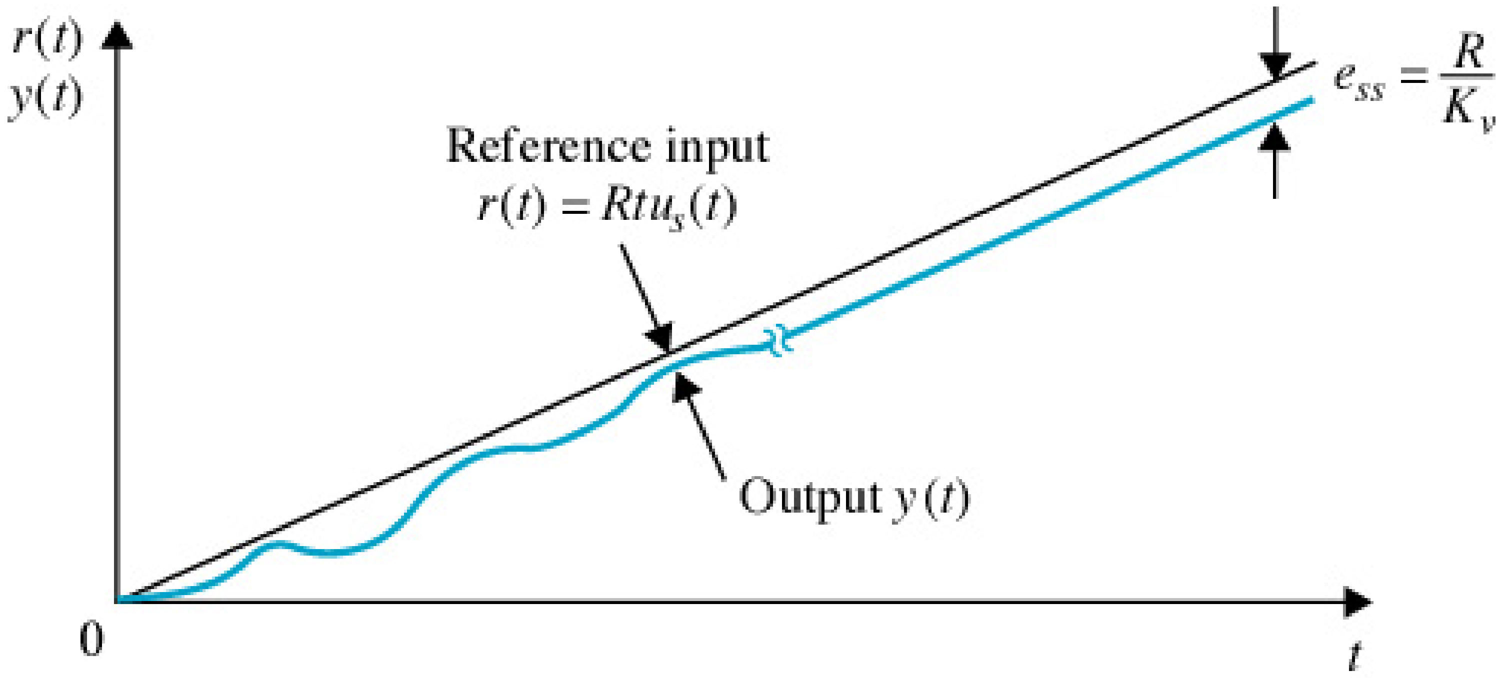
Unity feedback control system



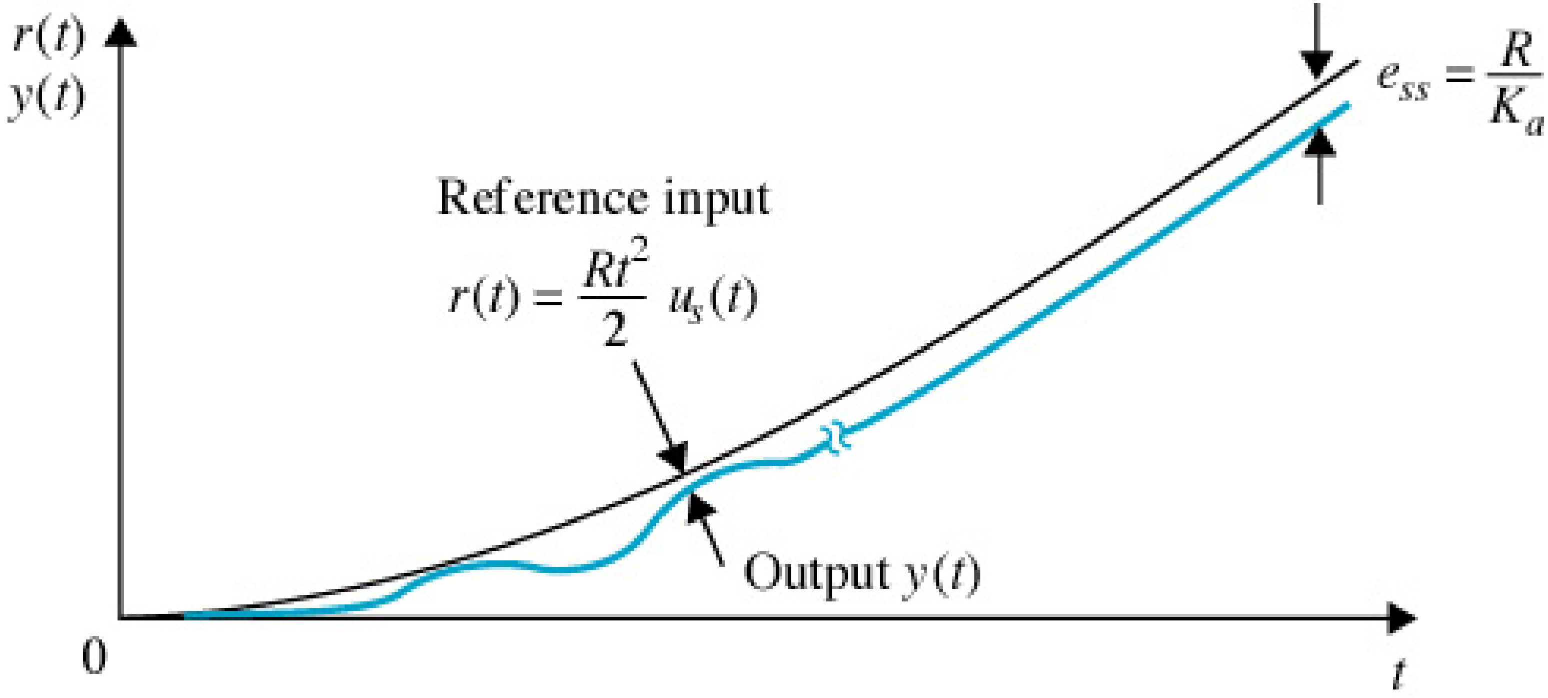
Steady state errors (Step Reference)



Steady state errors (Ramp Reference)



Steady state errors (Parabolic Reference)



Summary of steady state errors for different systems

		Type 0		Type 1		Type 2	
Input	Steady-state error formula	Static error constant	Error	Static error constant	Error	Static error constant	Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Type of system means the number of integrators that the system has in its loop transfer function.

Trade-offs between transient and steady-state performance

Increasing the number of integrators in the system can improve the steady-state performance of the system. However, because integrator has a pole at zero, it basically reduces the stability margin of the system.

Therefore, one could not improve one aspect of the system without paying due consideration to the other.